Variational Kalman Filter

Jean de Dieu Niyigaba

February 2012

Outlines

- Introduction
- Kalman filter algorithm
- VKF algorithm
- Word examples

Introduction to Kalman Filter

Definition

A Kalman filter is an optimal estimator, it infers parameters of interest from indirect, inaccurate and uncertain observations. It is recursive so that new measurements can be processed as they arrive.

...

Optimal in what sense?

If all noise is Gaussian, the Kalman filter minimises the mean square error of the estimated parameters.

Why use the word **FILTER?**

The process of finding the best estimate from noisy data amounts to However a Kalman filter also doesnt just clean up the data measurements, but also projects these measurements onto the state estimate.

...

What if the noise is NOT Gaussian?

Given only the mean and standard deviation of noise, the Kalman filter is the best linear estimator. Non-linear estimators may be better to use.

Basic linear kalman filter

Linear version of the general assimilation problem

$$\phi_k = M_k \phi_{k-1} + E_k \quad (1)$$

$$d_k = G_k \phi_k + e_k \quad (2)$$

where:

$\phi_k \in R^n$ is the state of the process at time k; d_k denotes the $mx1$ matrix of observed data; M_k is the nxn linear evolution matrix; E_k is a zero-mean Gaussian random vector of errors; G_k is the mxn linear observation matrix; $e_k \in R^m$ is a zero-mean Gaussian random vector of observed errors.

k denote the time index

The kalman filter algorithm (1/3)

step 0: Select initial guess ϕ_0^{est} and covariance C_0^{est} and set $k = 1$

Step 1:Compute the evolution model estimate ϕ_k^a and its covariance C_k^a:

A: Compute $\phi_k^a = M_k \phi_{k-1}^{est}$

B: Compute $C_k^a = M_k C_{k-1}^{est} M_k^T + C_{E_k}$

ϕ_{k-1}^{est} is the estimate of the state ϕ_{k-1} and C_{k-1}^{est} is the error covariance matrix of the estimate. C_{E_k} is the covariance of the prediction error E_k

The kalman filter algorithm (2/3)

step 2: Compute the Kalman filter estimate and covarince

A: Compute the kalman Gain $H_k = C_k^a G_k^T (G_k C_k^a G_k^T + C_a)^{-1}$

B: Compute the estimate $\phi_k^{est} = \phi_k^a + H_k(d_k - G_k \phi_k^a)$;

C: Compute the estimate covariance $C_k^{est} = C_k^a - H_k G_k C_k^a$

The kalman filter algorithm (3/3)

Step 3: Update $k := k + 1$ and return to step 1.

The Variational Kalman Filter Method

what is the use of this equivalent formulation of the Kalman filter?

The Variational Kalman Filter (VKF) method was introduced to compute the filter estimate and its covariance using an iterative minimization method.
This Important for large-scale problems where the exact Kalman filter is prohibitively expensive to compute.

The VKF algorithm (1/3)

Step 0: Select initial guess ϕ_0^{est} and covariance C_0^{est} and set k=1

Step 1: Compute the evolution model estimate and covariance:

A: Compute $\phi_k^a = \mathbf{M}_k \phi_{k-1}^{est}$

B: compute $\mathbf{C}_k^a = \mathbf{M}_k \mathbf{C}_k^{est} \mathbf{M}_k^T + \mathbf{C}_{\mathbf{E}_k} := \mathbf{C}_k^a$

The VKF algorithm (2/3)

Step 2: Compute the Variational kalman filter estimate and covariance:

A: compute the estimate $\phi_k^{est} = \arg\min_{\phi} \ell(\phi)$

with:

$$\ell(\phi) = \frac{1}{2}(d - \mathbf{G}_{\phi})^T \mathbf{C}_e^{-1}(d - \mathbf{G}_{\phi}) + \frac{1}{2}(\phi - \phi^a)^T (C^a)^{-1}(\phi - \phi^a) \tag{3}$$

B: compute the estimate covariance $C_k^{est} = \nabla^2 \ell(\phi)^{-1}$

where:

$\nabla^2 \ell(\phi) = G^T C_e^{-1} G + (C^a)^{-1}$ denotes the Hessian of ℓ

The VKF algorithm (3/3)

Step 3: Update $k := k + 1$ and return to step 1.

Why is Kalman Filtering so popular?

- Good results in practice due to optimality and structure.
- Convenient form for online real time processing.
- Easy to formulate and implement given a basic understanding.
- Measurement equations need not be inverted.

Word examples:

- Determination of planet orbit parameters from limited earth observations.
- Tracking targets - eg aircraft, missiles using RADAR.
- Robot Localisation and Map building from range sensors/ beacons.

Thank you!

MURAKOZE