Variational Ensemble kalman filtering technique in Jablonska-Capasso-Bianchi-Morale Model; Electricity and Oil spot Prices as case studies

Sihoja Hamis

Lappeenranta University of Technology

Feb 21, 2012

Outline

Jablonska-Capasso-Bianchi-Morale Model

$$dX_t^k = 3[\frac{\gamma_t}{3} X_t^* + \frac{\theta_t}{3} h(k, X_t) + \frac{\varepsilon_t}{3} g(k, X_t) - X_t^k]dt + \sigma_t dW_t$$

where $k = 1, 2, 3, \cdots, N$, stands for ensemble size,
γ_t is the moving mean reversion rate at the time t,
X_t^* is one year the moving average which model the short-term thinking of the traders about the market behavior
$\frac{\theta_t}{3} h(k, X_t)$ - Global momentum
$h(k, X_t)$ - Mean(Mean - Mode).
$g(k, X_t) - max(X_t^k - X_t)$

Least Square Method

$x_{t,t=1,2,3,...,n}$

$Y = b_0 + \sum_{t=1}^{k} b_t x_t$
b_0 and $b_{t,t=1,2,...,k}$
$\hat{b} = (X'X)^{-1}X'Y$
$cov(\hat{b}) = \delta^2(X'X)^{-1}$

Bayes formula

$\Pi(b) \propto P(Y/b)P(b)$

$P(Y/b)$ - is the likelihood function and
$P(b)$ - is the prior probability

Bayes Formula to Linear Kalman Filter

$$P(Y/b)P(b) = e^{-\frac{1}{2}((b-b_a)^{'} S_a^{-1}(b-b_a)+(Y-Xb)^{'} S_\epsilon^{-1}(Y-Xb))}$$

$S_a^{-1} = K_a^{'} K_a$ and
$S_\epsilon^{-1} = K_\epsilon^{'} K_\epsilon$
$-2 \log \Pi(b) = \parallel K_\epsilon X b - K_\epsilon Y \parallel^2 + \parallel K_a b - K_a b_a \parallel^2$

which is the norm of least square problem $\tilde{X} b = \tilde{Y}$ where $\tilde{X} = \begin{pmatrix} K_\epsilon X \\ K_a \end{pmatrix}$ and $\tilde{Y} = \begin{pmatrix} K_\epsilon Y \\ K_a b_a \end{pmatrix}$ with which least square method can be used to determine $\hat{b}$ and $cov(\hat{b})$ as $\hat{b} = (\tilde{X}' \tilde{X})^{-1} \tilde{X}' \tilde{y}$ and $cov(\hat{b}) = (\tilde{X}' \tilde{X})^{-1}$ which yield

$$\hat{b} = (X' S_\epsilon^{-1} X + S_a^{-1})^{-1} (X' S_\epsilon^{-1} Y + S_a^{-1} b_a)$$

$$cov(\hat{b}) = (X' S_\epsilon^{-1} X + S_a^{-1})^{-1}$$

Introduce $I = S_a^{-1} S_a$
To $\hat{b}$ and
$cov(\hat{b})$

$$
\begin{aligned}
cov(\hat{b}) &= (X' S_\epsilon^{-1} X + S_a^{-1})^{-1} S_a^{-1} S_a \\
&= (S_a X' S_\epsilon^{-1} X + I)^{-1} S_a \\
&= (S_a X' S_\epsilon^{-1} X + I)^{-1} ((S_a X' S_\epsilon^{-1} X + I) S_a - S_a X' S_\epsilon^{-1} X S_a) \\
&= S_a - (S_a X' S_\epsilon^{-1} X + I)^{-1} S_a X' S_\epsilon^{-1} X S_a \\
&= S_a - (X' S_\epsilon^{-1} X + S_a^{-1})^{-1} X' S_\epsilon^{-1} X S_a
\end{aligned}
$$

Taking S_ϵ in and S_a out, we end up getting
$cov(\hat{b}) = S_a - X^{'} S_a (X^{'} S_a^{-1} X + S_\epsilon)^{-1} X S_a$
let $G = X^{'} S_a (X^{'} S_a^{-1} X + S_\epsilon)^{-1}$
$cov(\hat{b}) = S_a - G X S_a$
Do the same for $\hat{b}$ to get
$\hat{b} = b_a + G(Y - X b_a)$

Kalman Filter for Nonlinear Models

- For simple nonlinear:linearize then use Extended Kalman Filter.
- For nonlinear with three parameters : Ensemble Kalman Filter.
- For nonlinear with four parameters : Variational Ensemble Kalman Filter.

Model fitting and results

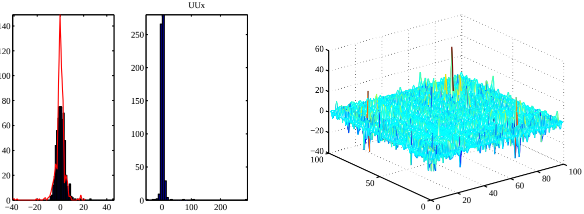


Figure: Original Prices with fitted in simulated electricity prices (left panel), mesh plot of simulated prices(right)

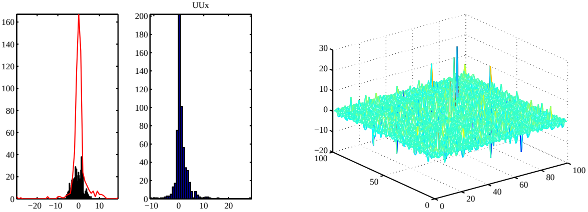


Figure: Original Prices with fitted in simulated oil prices (left panel), mesh plot of simulated prices(right)

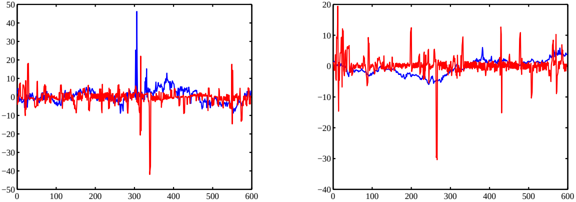


Figure: Timeline plot of original (blue) with simulated (red) for electricity and oil prices.

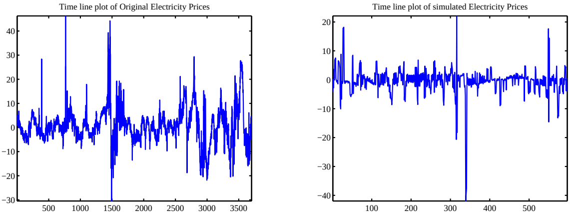


Figure: Timeline plot of original and simulated electricity prices.

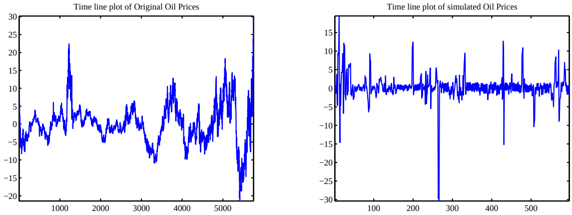
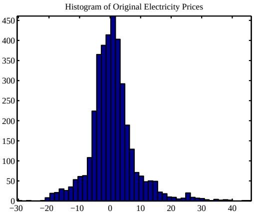


Figure: Timeline plot of original and simulated oil prices.



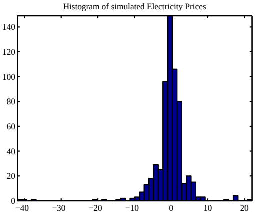


Figure: Histogram of original and simulated electricity prices.

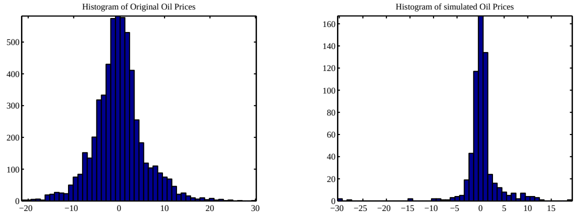


Figure: Histogram of original and simulated oil prices.

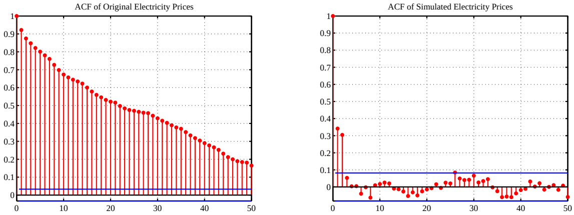


Figure: Autocorrelation plot of original and simulated electricity prices.

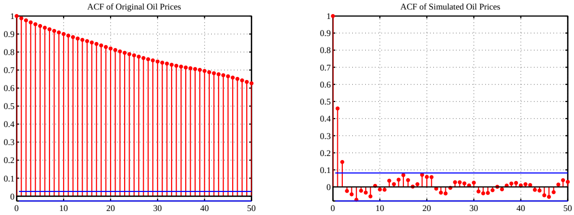


Figure: Autocorrelation plot of original and simulated oil prices.

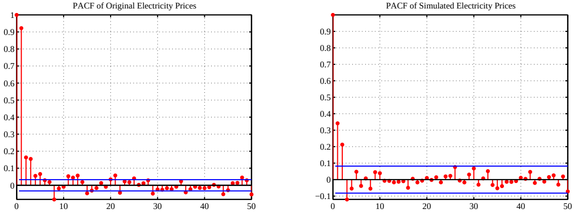


Figure: Partial Autocorrelation plots for Original and simulated Electricity prices.

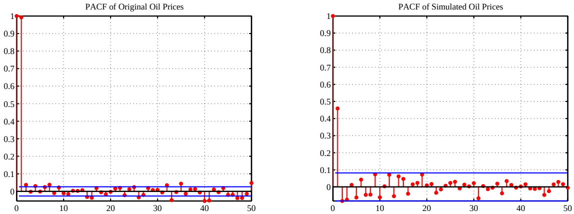


Figure: Partial Autocorrelation plots for Original and simulated oil prices.

	Original Elect prices	Simulated Elect prices
Mean	0.73	−0.40
Standard deviation	7.47	4.66
Skewness	0.92	−2.77
Kurtosis	6.97	30.4
5th moment	18.82	−212.5
6th moment	105	1929.96
7th moment	423	−15765.87

Table: Standard moments of Original electricity Prices and Simulated electricity Prices

Basic statistics

	Original Oil prices	Simulated Oil prices
Mean	0	0.29
Standard deviation	5.47	3.68
Skewness	0.3	−0.2
Kurtosis	4.86	26.64
5th moment	4.99	−166.57
6th moment	46.32	1484.19
7th moment	93.51	−11317.04

Table: Standard moments of Original oil Prices and Simulated Oil Prices

KIITOS