Statistacal Self-Similarity: *Fractional Brownian Motion*

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Outline

Hurst's Statistical Phenomenon

- It was introduced by a British hydrologist H. E. Hurst in 1951.
- Discovery of Hurst's parameter or exponent

Definition of Statistical Self-Similarity

Two processes are said to be self-similar if their joint distributions are identical.

Typical example of self-similar processes is a well known Brownian motion. This process have independent increments.

Background and Definition of FBM

- Fractional Brownian Motion (FBM) belongs to a class of long-memory Gaussian process
- FBM is a generalization of the more well-known process of Brownian motion
- The dependence structure of the increments is modeled by a parameter $H \in (0, 1)$

A standard Fractional Brownian Motion with Hurst parameter $0 < H \leq 1$ is a continuous Gaussian process $B = (B_t) \geq 0$ with zero mean and a covariance function given

$$cov(B) = EB_sB_t = \frac{1}{2}\{|s|^{2H} + |t|^{2H} - |t-s|^{2H}\}$$

Basic Properties of FBM

- $B_H(0) = 0$ and $EB_H(t) = 0 \Rightarrow$ stationary increments
- Self-similar

Antipersistent

When the parameter $0 < H < 1/2$, The increments are said to be antipersistent. They are said to have short-range dependence or memory.

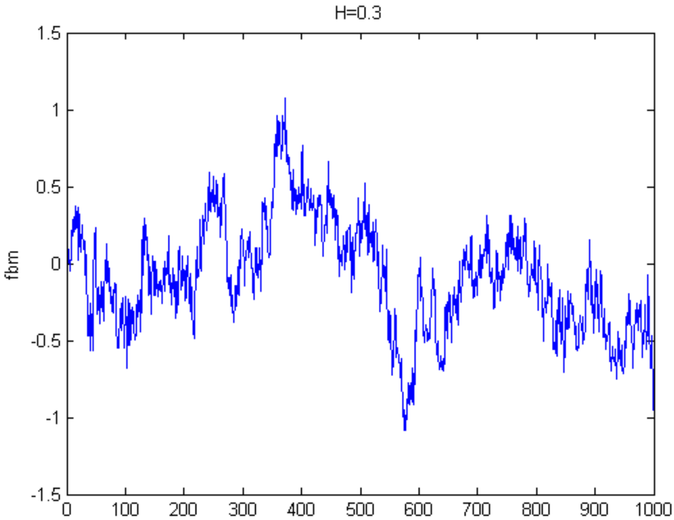

Figure: $0 < H < 1/2$: Antipersistent

Independent Increments

When the parameter $H = 1/2$, the process is said to have independent increments. It corresponds to the standard Brownian motion.

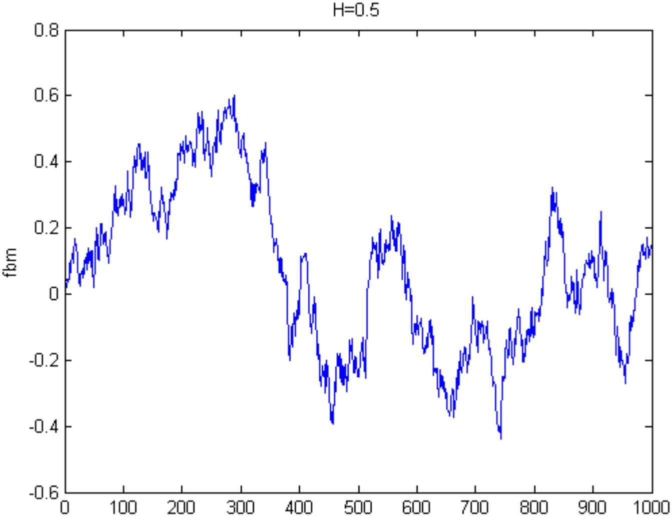

Figure: $H = 1/2$: Independent increments

Persistent

When the parameter $1/2 < H < 1$, The increments are said to be persistent. They are said to have long-range dependence or memory.

The increments are positively correlated

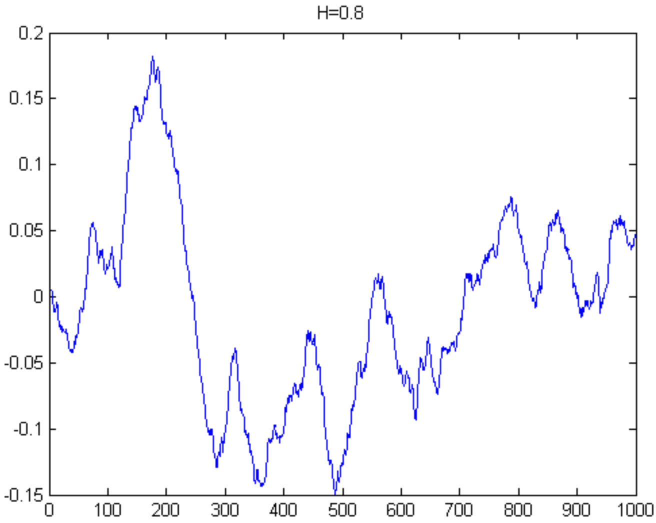

Figure: $1/2 < H < 1$: Persistent.

Role of Hurst's Parameter H

Summary

It determines the sign of the covariance of the future and past increments

Existence of long-range dependence

Background

- Model due to Cajueiro and Barbachan (2003)
- To study the behaviour of Brazilian stock returns in continuous time
- Price European options using Black-Schole type formula

The Market Model

- We consider the fractional Black-Scholes market
- The investor is confronted by two investment options

Risk-free asset: price given by
$dB(t) = \rho B(t)dt \quad [B(0) = 1, 0 < t \leq T, \rho > 0]$

Risk asset: price given by $dS(t) = \mu S(t)dt + \sigma S(t)dB_H(t) \quad [S(0) = s_0]$

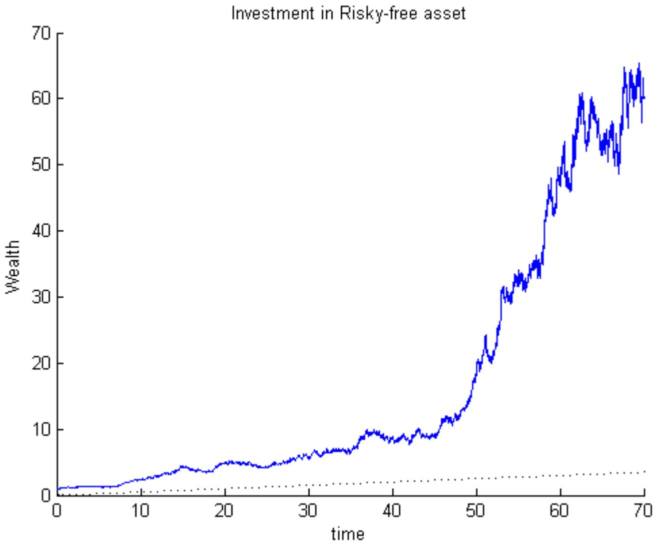

Figure: Investment in risk-free asset: the wealth grows exponentially

Option Pricing Formula

We consider the fractional Black-Scholes formula, where the total wealth is given by

$$C = S(t)N(d_1) - Ke^{-r(T-t)}N(d_2)$$

Sample results

$3 - D$ plot for FBS - BS call option, Maturity time and Exercise price

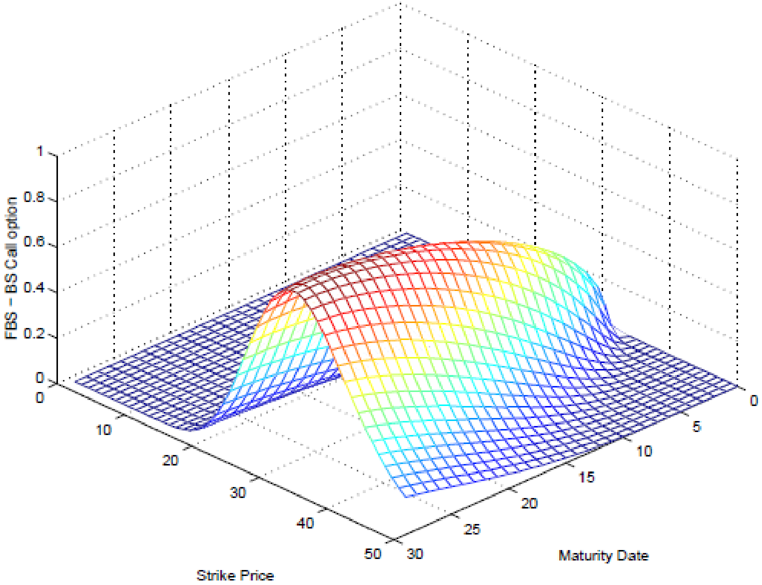

Figure: FBS-BS, Maturity and Exercise Price

THANK YOU