A non-Gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing

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16,April 2009

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INTRODUCTION

Futures contracts are contracts to buy or sell at a specific date in the future at a price specified today. The future date is called the delivery date or final settlement date.

F**orward contract** is an agreement between 2 parties to buy or sell an asset at a specified point of time in the future. The price is paid before control of the instrument changes.

Contracts terms are set now but delivery and payment will occur at a future date.

Spot price is the price that is quoted for immediate (spot) settlement (payment and delivery).

The **spot market** or **cash market** is a commodities or securities market in which goods are sold for cash and delivered immediately.

Derivative is a contract whose value is derived from that of other quantities.

Ornstein–Uhlenbeck process also known as the mean-reverting process, is a stochastic process *r*given by the following stochastic differential equation:

$$dr_t = \theta(\mu - r_t)\,dt + \sigma\,dW_t.$$

where $\theta > 0$, μ and $\sigma > 0$ are parameters and *Wt* denotes the wiener process.

INTRODUCTION

AIM

- We propose a mean-reverting model for the spot price dynamics of electricity which includes seasonality of the prices and spikes.
- The dynamics is a sum of non-Gaussian O-U processes with jump processes giving the normal variations and spike behaviour of the prices.
- We demonstrate in a simulation example that the model seems to be sufcientIy flexible to capture the observed dynamics of electricity spot prices.

Two categories of approaches for electricity price modelling:

1. Direct modelling of futures prices

- Transfer of concepts from interest rate theory.
- Advantage: complete market and risk neutral pricing machinery available
- Problem: no inference about spot prices possible (arbitrage relations not valid)

2. Spot price modelling

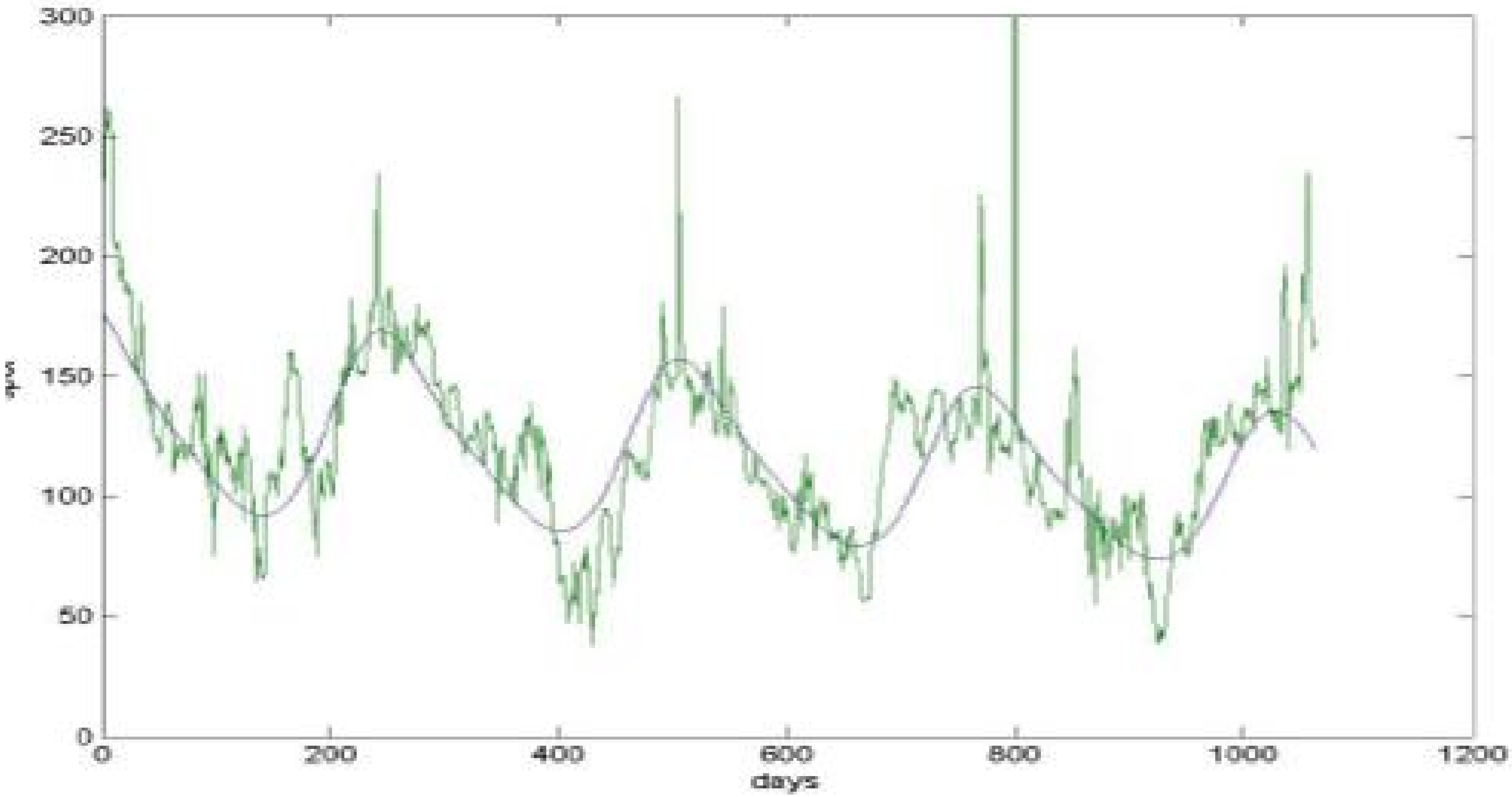
- Various structured OTC products depend on spot evolution → spot price model required

- Use spot price model to derive prices of futures (and other derivatives)

- breakdown of spot-futures relationship → identification of market price of risk (pricing measure) necessary to derive futures prices

Essential features of spot prices

Daily NordPool system price 01.1997-01.2001



Essential features of spot prices

Stylized features of electricity spot prices are:

- Mean reversion
- seasonality
  - yearly price cycle (in example above winter has higher prices than summer)
  - weekly seasonality
  - intra-daily cycles
- intrinsic feature of sharp spikes followed by sharp drops
- intrinsic feature of sharp spikes followed by sharp drops

Modelling requirements of spot dynamics

A spot price model should

- reflect statistics and path properties of historical data
- reflect physical conditions and constraints
- but also allow for sufficient analytical tractability:

  -risk evaluation

  -forward/futures price dynamics

  -option pricing

In particular, analytical pricing of forwards and futures is very desirable.

Common spot price models

Most common reduced form spot price models are of exponential Ornstein-Uhlenbeck type

- guarantees positive prices
- enhances robustness of calibration procedure

However

- Is the exponential structure the right transformation for electricity prices?

  exponential structure originates from population growth modelling (in finance compound interest modelling)
- Most importantly, no manageable analytic expressions for corresponding forward/futures contracts!

An arithmetic model

We propose to model the spot price as a sum of non-Gaussian OU-processes:

$$S(t) = \Lambda(t) + \sum_{i=1}^{n} Y_i(t)$$

where

$$dY_i(t) = -\lambda_i Y_i(t)\, dt + \sigma_i(t)\, dL_i(t)$$

- $L_i(t)$ are independent increasing time inhomogeneous pure jump Lévy processes (additive processes).
- We suppose a Lévy measure of $L_i(t)$ of the form

$$\nu_i(dt, dz) = \rho_i(t)\, dt\, \nu_i(dz)$$

where $\rho_i(t)$ controls seasonal variation of jump intensity.

An arithmetic model

- $\rho_i(t)$ controls seasonal variation of jump sizes
- λ_i different level of mean reversion
- $\Lambda(t)$ deterministic seasonality function

$\rightarrow$ The model guarantees positive prices because the $L_i(t)$ are increasing.

$\rightarrow$ Upward jumps are followed by downward drops whose sharpness is controlled by the corresponding λ_i

$\rightarrow$ The model allows for analytical pricing of corresponding forward and fututres contracts.

Pricing of forward/futures contracts

- Let $F(t; T_1, T_2)$ be time t forward price of a contract which delivers electricity at a rate $S(t)/T_2 - T_1$ during the settlement period $[T_1, T_2]$:

$$\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)\, du .$$

- Forward price defined so that time t value is zero, given information about the spot price up to time t:

$$F(t; T_1, T_2) = \mathbb{E}_Q \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(u)\, du \,|\, \mathcal{F}_t \right],$$

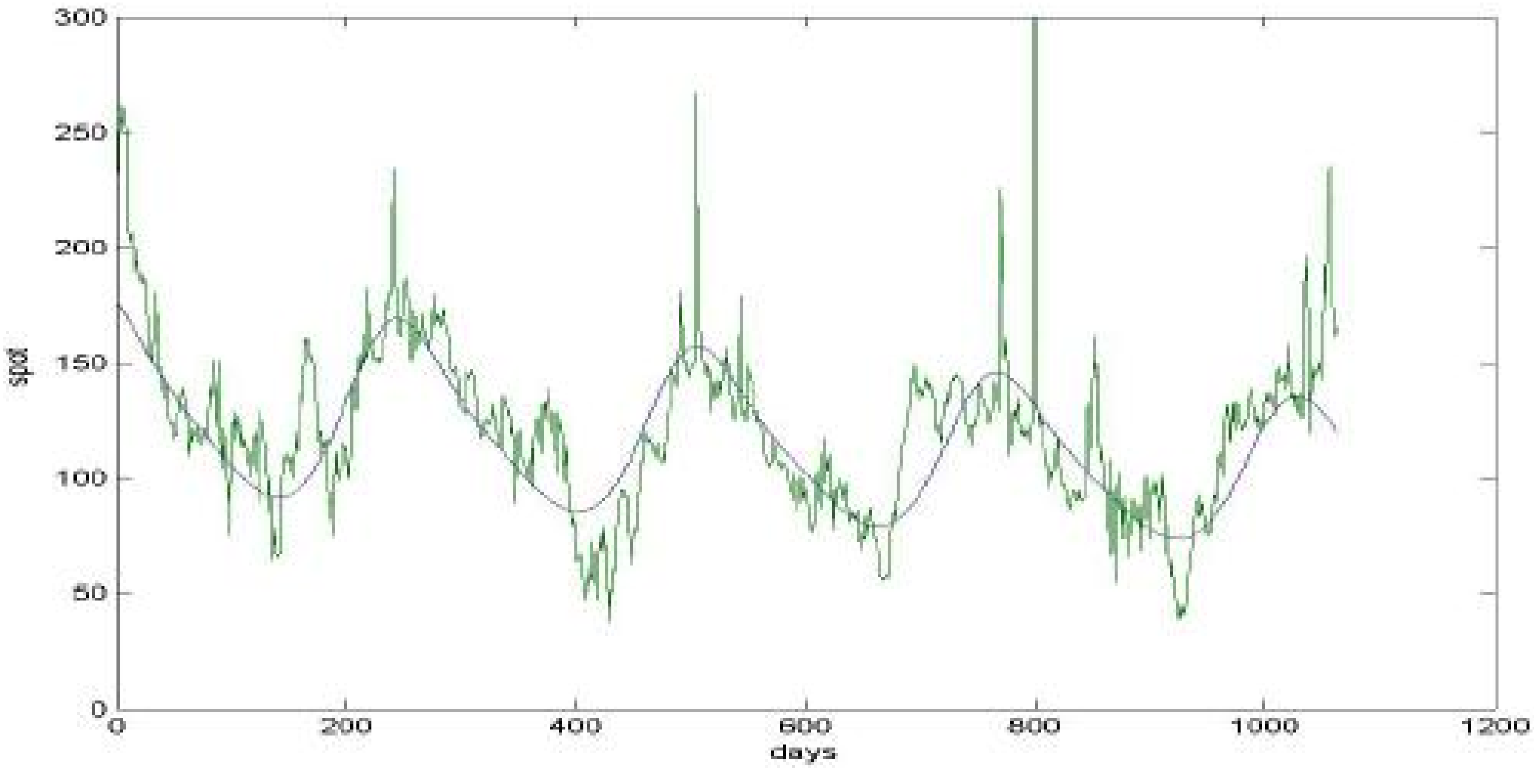
- where Q is a pricing measure to be determined.

Pricing of options on forward/futures contracts

- Let $g \in L^1(\mathbb{R})$ be payoff of an option written on $F(T;T_1,T_2)$, $T \leq T_1$. Then the price is given by

$$p(t;T;T_1,T_2) = e^{-r(T-t)}\mathbb{E}_Q\left[g(F(T;T_1,T_2)) \,|\, \mathcal{F}_t\right].$$

Case study: simulation of the NordPool spot



We want to fit the model

$$S(t) = \Lambda(t) + \sum_{i=1}^{n} Y_i(t)$$

$$dY_i(t) = -\lambda_i Y_i(t)\,dt + \sigma_i(t)\,dL_i(t)$$

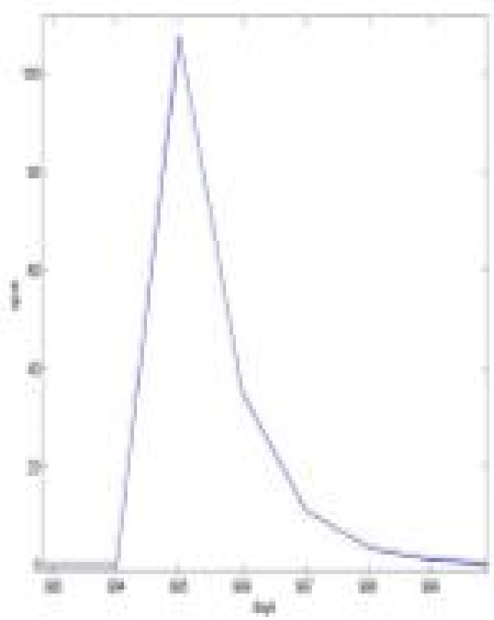
Case study: simulation of the NordPool spot

In order to fit the model to the time series of daily Nordpool spot price given above, we proceed in four steps:

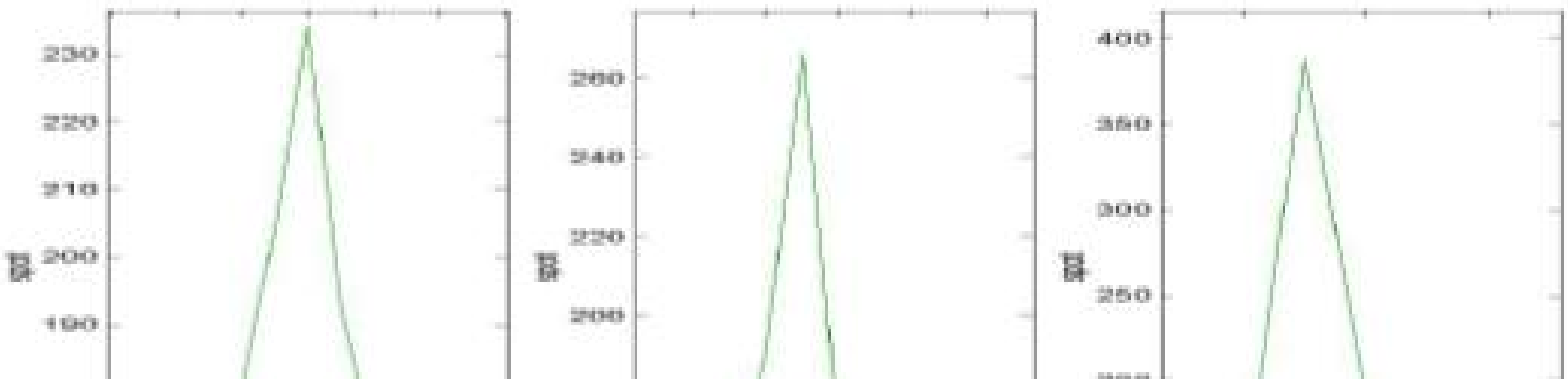
1. Identification of the first OU-process $Y_1(t)$ modelling the seaonal spikes.
2. We remove the spikes from the spot series and fit a deterministic seasonal mean $\Lambda(t)$ of cosines to the remaining time series.
3. We remove the seasonal mean and fit a sum $\sum_{i=2}^{n} Y_i(t)$ of stationary OU-processes to the remaining time series.
4. Simulation of a sample path.

1. Identification of Y1(t) modelling the seaonal

Simulated spike:



Zoom into the 3 biggest spikes of 1998, 1999, 2000:



Case study: simulation of the NordPool spot

- For an estimated mean reversion $\hat{\lambda}_1 = 1.12$ (2/3 decay after one day), find the OU-path $\hat{Y}_{1(t)}$ with jump times $\hat{\tau}_1 \leq \ldots \leq \hat{\tau}_{\hat{r}}$ and corresponding path values $\hat{\mu}_1, \ldots, \hat{\mu}_{\hat{r}}$ that minimize

$$\min_{\substack{1 \leq \tau_1 \leq \ldots \leq \tau_r \leq N \\ \mu_1, \ldots, \mu_r \\ r \in \{1, \ldots, N\}}} \left\{ \gamma \cdot r + \sum_{i=1}^{r+1} \sum_{t=\tau_{i-1}}^{\tau_i - 1} \left(spot(t) - \mu_{\tau_{i-1}} e^{-\hat{\lambda}(t - \tau_{i-1})} \right)^2 \right\}$$

- γ represents penalization of jumps

- Using dynamic programming, an adoption of an algorithm from (Winkler, Liebscher) yields an exact algorithm to solve the above min-problem.

Case study: simulation of the NordPool spot

- We assume the first OU-component given through $Y_1(t)$

	λ	$\sigma(\mathbf{t})$	$\nu(\mathbf{dz})$	$\rho(\mathbf{t})$
Y_1	1.12	1	Exp(180)	$0.07 \cdot \left(\frac{2}{\left\| \sin\left(\frac{\pi(t-6)}{261}\right) \right\| + 1} - 1 \right)$

Case study: simulation of the NordPool spot

2. We fit a deterministic seasonal mean $\Lambda(t)$ of cosines to the time series $spot(t) - \hat{Y}_1(t)$.
3. We de-seasonalize the spot price process by removing seasonal spikes and deterministic mean level:

$$despot(t) = spot(t) - \Lambda(t) - \hat{Y}_1(t),$$

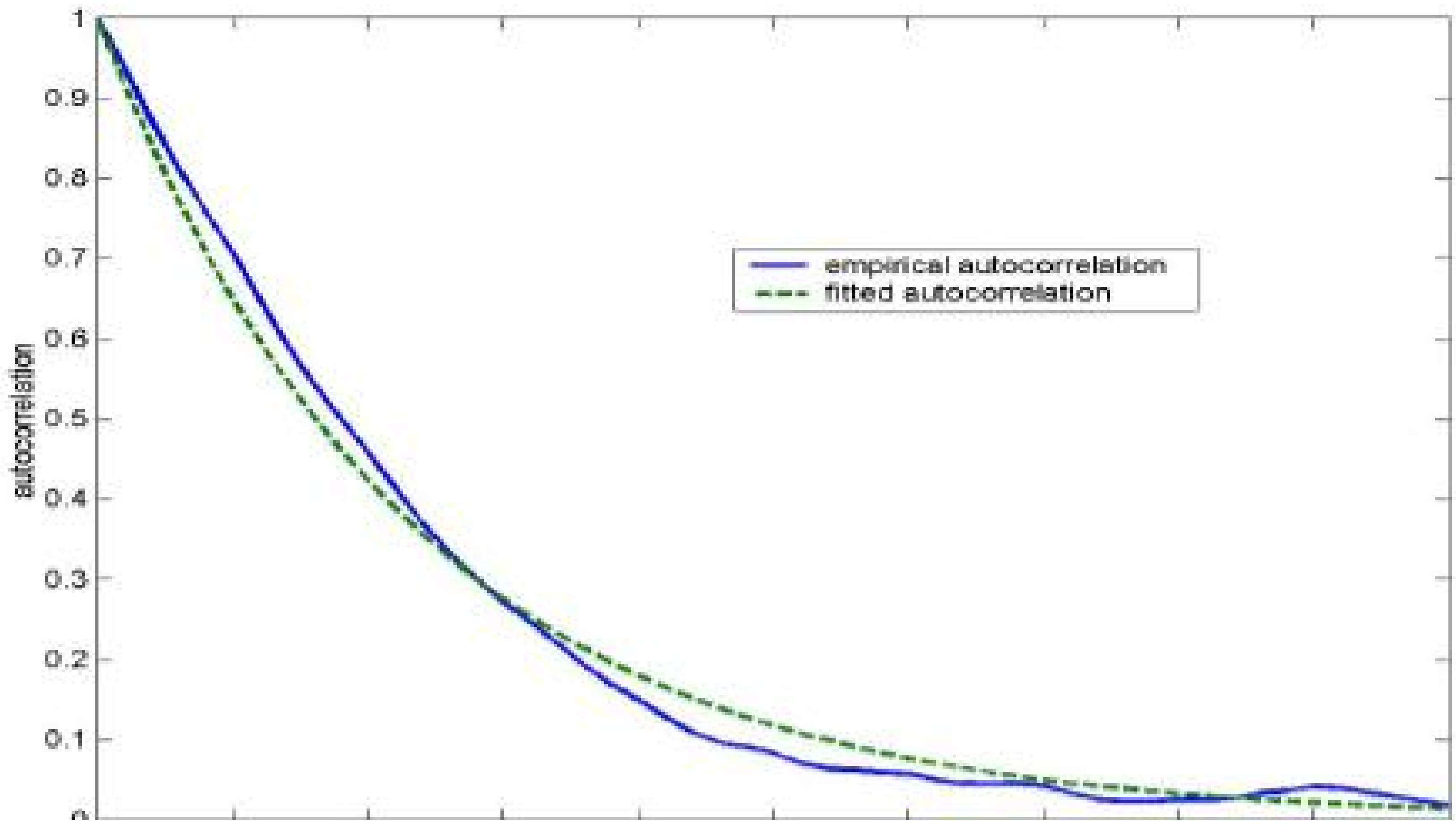
and calibrate a sum of stationary OU-processes to the de-seasonalized spot price:

$$X(t) := \sum_{i=2}^{n} Y_i(t) \sim despot(t),$$

where $dY_i(t) = -\lambda_i Y_i(t)\,dt + dL_i(t)$ with now L_i increasing Lévy processes (no variation over time in controls).

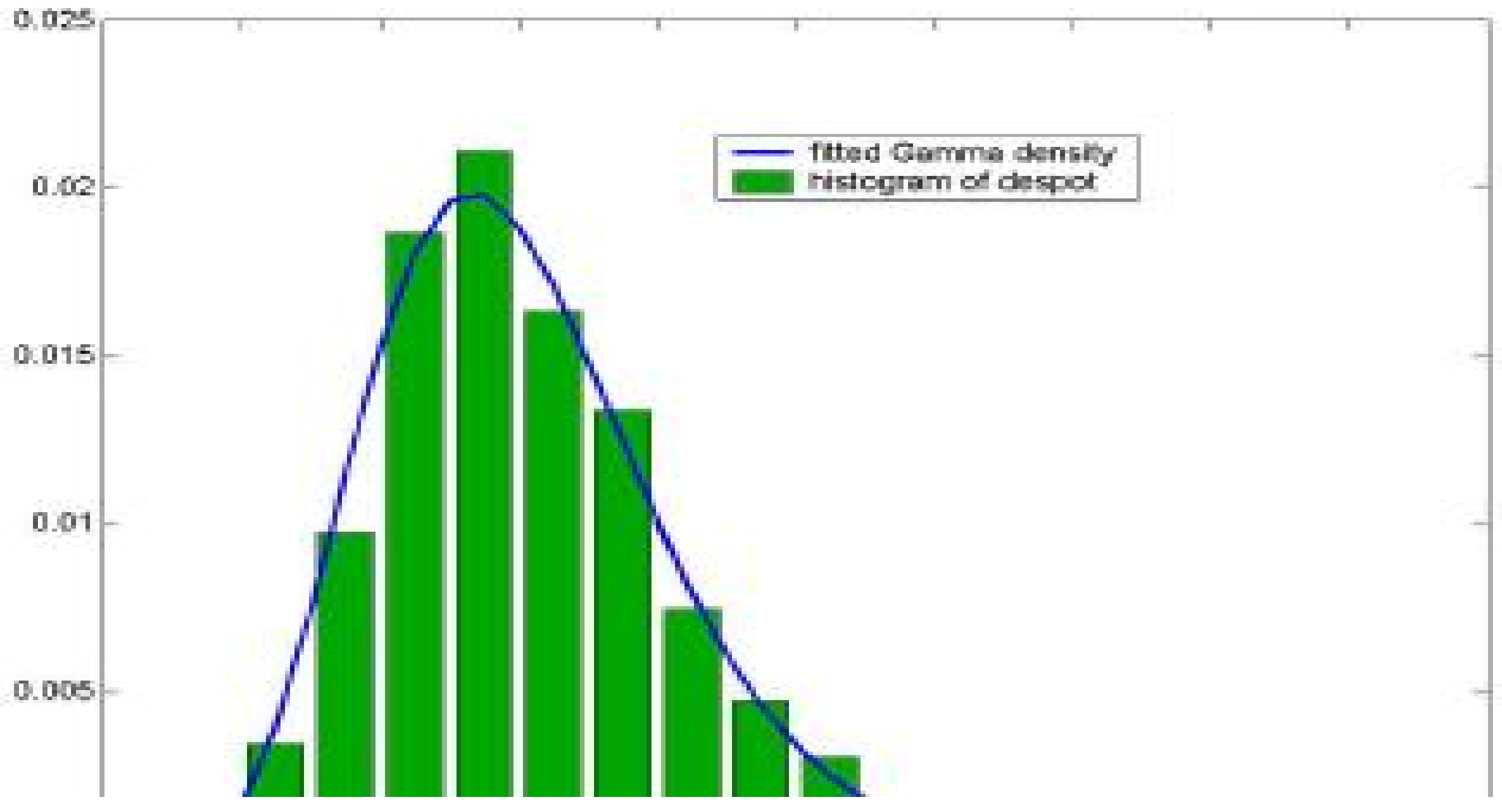
Case study: simulation of the NordPool spot

- Already one component $X(t) = Y_2(t)$ with $\hat{\lambda}_2 = 0.0846$ is sufficient to optimally fit the empirical autocorrelation structure:



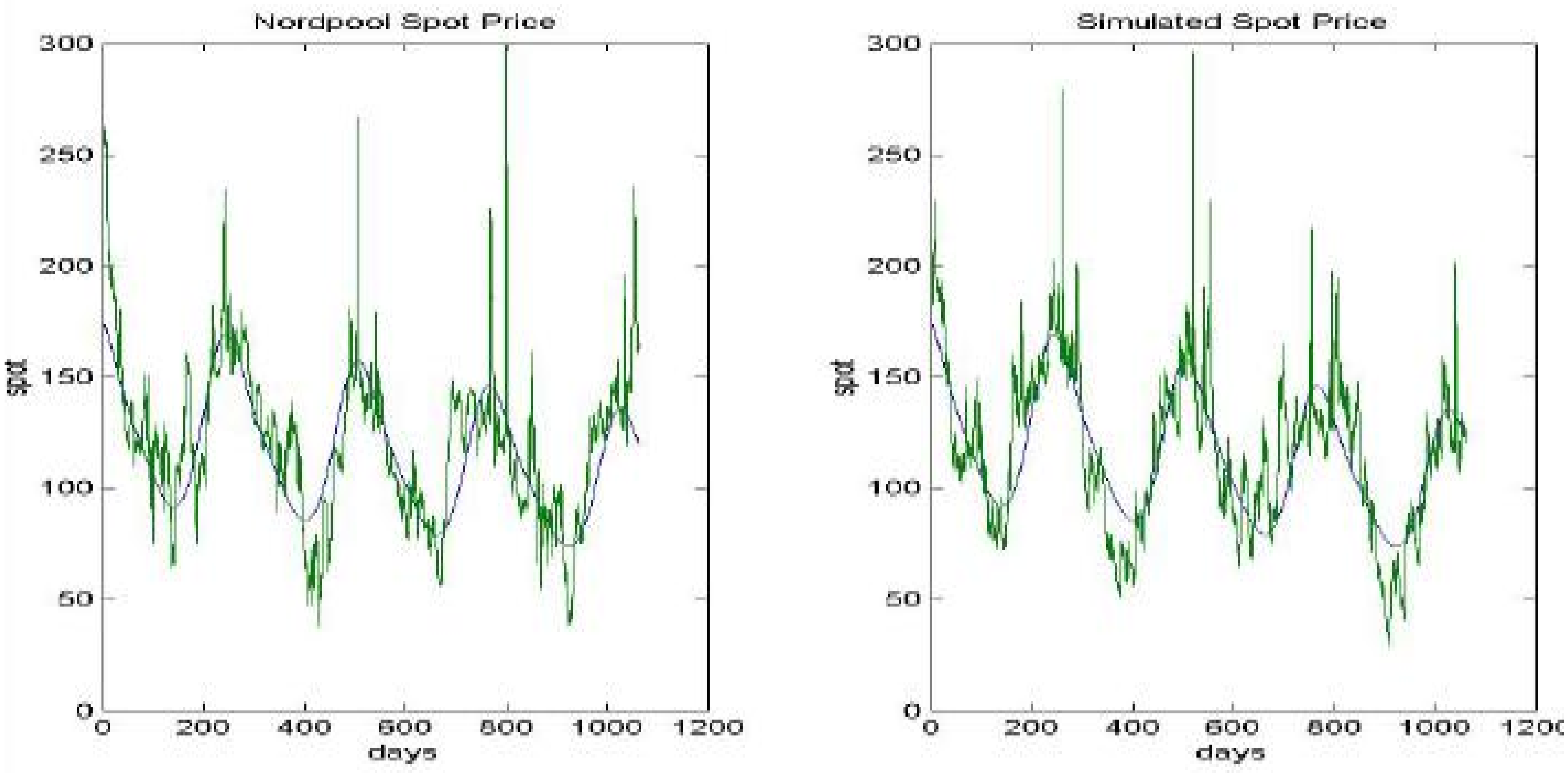
Case study: simulation of the NordPool spot

- We assume $Y_2 \sim \mathrm{Gamma}(\nu, \alpha)$ and estimate $\nu = 8.055$, $\alpha = 0.132$ through performing maximum likelihood on despot:



Case study: simulation of the NordPool spot

4. Simulation of a complete path of the estimated process $S(t) = \Lambda(t) + Y_1(t) + Y_2(t)$:



Case study: simulation of the NordPool spot

- Empirical moments of NordPool spot price versus simulated moments (averaged over 3000 simulation paths):

	Mean	Std. Dev.	Skewness	Kurtosis
Empirical	121.2387	35.9166	0.8516	6.7061
Simulated	120.6239	36.4370	0.8276	6.4120

Conclusion

- Most common spot models are of geometric type and become unfeasible for further analysis of derivatives pricing.

- We propose an arithmetic model that is simple enough to yield analytical forward prices. Option pricing by fast Fourier transform techniques.

- The arithmetic model describes well both path properties and statistics of electricity spot prices.

- Future work includes the calibration of the market price of risk and the study of futures prices induced by the model.

MURAKOZE!