Capasso and Morale: Asymptotic behavior of a system of stochastic particles subject to nonlocal interactions

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Spring 2009

Overview

Overview

1. Data Sets
2. Modelling approaches
3. A central challenge!
4. An example of mesoscale interaction
5. The Capasso-Morale model

Data Sets

- Electricity spot market prices and background data

Overview

Classical time series forecasting

- Box-Jenkins analysis, in which the time series is reconstructed from
- A polynomial (AR) fitted to low-pass filtered past values (MA)
- Generated from a normally distributed stochastic noise process
- AR, MA, ARMA, ARIMA, ARCH, GARCH,...

Stochastic differential equations (SDEs)

- Ordinary differential equations in which the solution is **distribution-valued (or probability measure valued)**
- Individual solutions are stochastic processes
- But the solution mean follows a deterministic path
- When an SDE is linear, this corresponds to normally distributed noise source
- A special course on SDEs starting soon by Prof. W. Charles Mahera!

Vector valued filtering

- The most recent values of a time series constitute a state vector
- A state vector can also be formed by feature extraction from past values
- Low-pass or band-pass filtering by e.g. following the autocorrelation function is an example of feature extraction
- Therefore different version of Kalman filter, Neural networks, Genetic algorithms can be applied

Lagrangian or Ensemble or MCMC methods

- Lagrangian methods follow individual trajectories and
- Compile posterior solution distributions from these by variants of Bayes' formula
- Examples: MCMC, Particle filtering, Ensemble forecasting

Overview

Three-level dynamics in time series

- *Micro-scale*: individual instances of a stochastic process
- *Macro-scale*: the deterministic dynamics of the parameters of solution distributions
- *Meso-scale*: the interdependence between micro-scale and macro-scale
- Meso-scale is non-trivial if dynamics are non-linear!

Meso-scale interaction

- Micro-scale transition probability depends on macro-scale distribution - **a field**
- Macro-scale distribution represents micro-scale solution density - **aggregated by MCMC**
- Meso-scale: What if micro-scale transition probability depends on a **band-passed semi-local density field**?
- This is the case in fluid mechanics and causes chaoticity, i.e. turbulence
- A typical example: collective or herding behavior in financial time series
- This phenomenon is caused by **limited** - **but still non-local** - **visibility**

Our research program on electricity time series

- Are price spikes caused by regime transitions? (constrained vs. unconstrained market)
- Or are they caused by price herding?
- The latter would explain why we fail in predicting financial chaos
- We can find this out by analyzing our time series data!

The Efficient Market Hypothesis

- States that liquid markets (=the law of large numbers applies) always price tradable objects right
- This means that price shifts are caused by external changes only, e.g. changes in supply or demand
- It is equivalent to assuming that all traders behave rationally and have complete information
- Corresponding SDEs are linear and price and return variances normally distributed, corresponding to Brownian motion
- Corresponding PDEs are linear and parabolic

The Big Question

Quest

Can we show to what extent spikes in price returns disobey the Efficient Market Hypothesis (and thereby prove Hyman Minsky right)?

- In Minsky's world there is limited information and herding behavior among traders
- The corresponding SDEs and PDEs have interaction between micro- and macro-level, their dynamics are not Brownian
- Corresponding PDEs are nonlinear (because of herding) and hyperbolic (because of limited visibility)
- Residual series exhibit long memory and fat tails

Overview

An example of meso-scale interaction - 1

- A home buyer in Reykjavik
- Sees housing prices rice steadily in Reykjavik, or in her suburb, for a decade - **with limited visibility at suburb level**
- She takes a big mortgage to get a dream home before its price rises beyond her reach
- Her bank grants it in pounds because they have good access to capital markets in London and the interest is lower there

An example of meso-scale interaction - 2

- Her bank bundles such mortgages with good debtors and tops them up with a big loan to
- Invest in American CDOs (Collateralized Debt Oblications) that have been doing well on Wall St for five years
- CDOs mix good and bad mortgages to spread their risk and used to get investment class rating - **limited visibility at Wall St. financial markets level**

An example of meso-scale interaction - 3

- All big banks do similar assessments and buy CDOs worth trillions
- Eventually all profits accumulated at all levels create such an abundance of capital that there is nowhere to invest that any more with a positive expected return
- Some banks start selling CDOs short to get out before the deluge
- This process is contagious - and fast - **with limited visibility at Wall St.**

An example of meso-scale interaction - 4

- Our home buyer's bank in Iceland hears the bad news last and loses a lot because of its big debt
- That big debt is called and the bank cannot get any more capital
- No Icelandic bank can give out mortgages any more because they have overextended their financial resources
- Nobody gets a mortgage any more in Iceland
- Therefore houses cannot be sold and their prices plummet because of oversupply
- The value of the home our buyer bought is less than her remaining mortgage

An example of meso-scale interaction - 5

- Too large a proportion of Iceland's economy hinges on the banking sector
- The value of the Icelandic krona plummets
- The total value of houses in Iceland is less than the debt in pounds/euro/dollar that its banks and mortgage takers own
- Iceland has lost all its property abroad and must ask for emergency help from IMF
- People blame their governement - **limited visibility** - and take to the streets

Overview

Capasso-Morale model

- Describes all three levels of interactions as coupled SDEs/ODEs
- Features Lagrangian description at micro-scale and
- Eulerian description at macro-scale and
- Interaction potentials in meso-scale - *aggregation* at macro-scale boundary and *repulsion* at micro-scale boundary

Lagrangian micro-scale description

- by an Itô type SDE for every particle:

Equation

$$dX_N^k(t) = \mathcal{H}_N^k(X_N^1(t), \ldots, X_N^k(t), t)dt + \sigma dW^k(t), \quad k = 1, \ldots, N$$

Eulerian macro-scale description

- by aggregating the trajectories of all particles:

Equation

$$X_N(t) = \frac{1}{N}\sum_{k=1}^{N} \epsilon_{X_N^k(t)} \in \mathcal{M}_P(R^d)$$

- where ϵ represents the spreading kernel in state space R^d of the state a single particle

With a meso-scale interaction potential

Equation

$$\mathcal{H}_N^k(X_N^1(t), \ldots, X_N^k(t), t) = \nabla G * X_N(X_N^k(t)) - \nabla V_N * X_N(X_N^k(t))$$

- where G is a Vlasov long range kernel and V_N is a moderate short ranged kernel
- * means convolution: $G * X(x) = \int G(x-s)X(s)ds$

Properties of Capasso-Morale solutions

- The macro-scale solution of the system is a probability density field - i. e. *a limit measure*
- This density is a solution to a deterministic integro-differential equation
- The density describes the mean-field spatial density of particles in the population
- In financial time series, this is density on a price-time or returns-time (i.e. log(price)-time) plane

The deterministic integro-differential equation

Equation

$$\frac{\partial}{\partial t}\rho(x,t) = \nabla \cdot (\rho(x,t)\nabla\rho(x,t)) - \nabla \cdot (\rho(x,t)\nabla G * \rho(\cdot,t))(x)),$$

$x \in R^d, t \geq 0$

- where the second term with $\nabla G*$ is the integro- part
- The divergence operator $\nabla \cdot$ aggregates a vector into a scalar density field ρ
- Note the "'momentum term"' $\rho(x,t)\nabla\rho(x,t)$!

The deterministic integro-differential equation and Navier-Stokes equations

- Note the "'momentum term"' $\rho(x,t)\nabla\rho(x,t)$!
- This is the same term as in Navier-Stokes equations (i.e. in 1D Burgers' equation)

Equation

$$\frac{\partial}{\partial t}u(x,t) = u(x,t)\nabla u(x,t))$$

- Burgers' equation starts from a sine wave and produces in finite time a saw-tooth wave
- This describes the slow inflation of an asset bubble, followed by a rapid burst

Properties of Capasso-Morale densities

- The Capasso-Morale integro-differential equation is degenerate
- It does not generally have unique solvability
- Consequently it does not admit an entropy solution: its drift (=the path taken in state space by its mean) is not bounded uniformly in N!
- A nontrivial invariant limit measure therefore does not exist
- Under some conditions (e.g. by adding diffusion) this can be mended
- But we do not know if these conditions are fulfilled in real financial markets!

Our Quest

Quest

In your Master and PhD Theses, please find this out in the case of electricity spot markets!