INTRODUCTION TO KALMAN FILTERING

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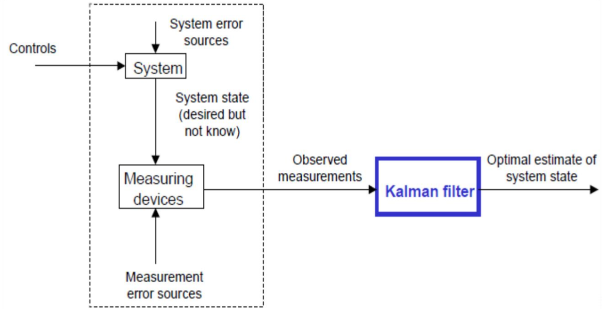
Outline

- Introduction to the concept
- Which is the best estimate?
- Basic Assumptions
- The meaning of the error covariance matrix

Introduction to the concept

- Kalman Filter (KF) is an Optimal Recursive Data Processing Algorithm.
- Typical Kalman filter application



Introduction to the concept

Optimal ...

- Dependent upon the criteria chosen to evaluate performance.
- Under certain assumptions, KF is optimal with respect to virtually any criteria that makes sense.
- KF incorporates all available information
  - knowledge of the system and measurement device dynamics
  - statistical description of the system noises, measurement errors, and uncertainty in the dynamics models
  - any available information about initial conditions of the variables of interest

Introduction to the concept

Optimal ...

$$x(k+1) = f(x(k), u(k), w(k)) \tag{1}$$

$$z(k+1) = h(x(k+1), v(k+1)) \tag{2}$$

Where,

- x - state
- f - system dynamics
- u - controls
- w - system error sources
- z - observed measurements
- h - measurement function
- v - measurement error sources

Introduction to the concept

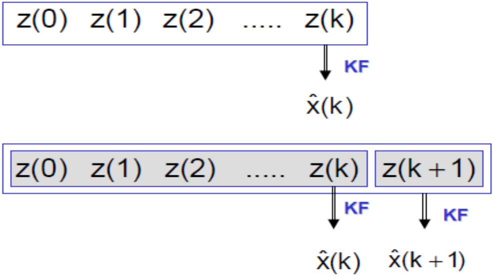
Optimal ...

- Given
  - f, h, noise characterization, initial conditons
  - $z(0)$, $z(1)$, $z(2)$, . . . , $z(k)$
- Obtain
  - the best estimate of $x(k)$

Introduction to the concept

.... Recursive ...

- The KF does not require all previous data to be kept in storage and reprocessed every time a new measurement is taken.



Introduction to the concept

.... Recursive ...

- To evaluate $\hat{x}(k+1)$,
  the KF only requires $\hat{x}(k)$ and $z(k+1)$.

Introduction to the concept

.... **Data Processing**

- The KF is a data processing algorithm.
- The KF is a computer program running in a central processor.

Which is the best estimate?

- Any type of filter tries to obtain an **optimal** estimate of desired quantities from data provided by a noisy environment.
- Best: minimizing the mean square error of the estimated parameters.
- Bayesian viewpoint - the filter propagates the **conditional** probability density of the desired quantities, conditioned on the knowledge of the actual data coming from measuring devices.

Which is the best estimate?

Example

- $x(i)$ one dimensional position of a vehicle at time instant i.
- $z(j)$ two dimensional vector describing the measurements of position at time j by two separate radars.
- If $z(1) = z_1, z(2) = z_2, \ldots, z(j) = z_j$

$$P_{x(i)|z(1),z(2),\ldots,z(j)}(x|z_1, z_2, \ldots, z_j)$$

  - represents all the information we have on $x(i)$ based (conditioned) on the measurements acquired up to time i.
  - given the value of all measurements taken up to time i, this conditional pdf indicates what the probability of $x(i)$ would be assuming any particular value or range of values.

Which is the best estimate?

- The shape of $P_{x(i)|z(1),z(2),\ldots,z(j)}(x|z_1, z_2, \ldots, z_j)$ conveys the amount of certainty we have in the knowledge of the value x.
- Based on this conditional pdf, the estimate can be:
  - the mean - MMSE(minimum mean square error).
  - the mode - MAP (maximum a posterior).
  - the median - the value of x such that half the probability weight lies to the left and half to the right of it.

Basic Assumptions

- The Kalman Filter performs the conditional probability density propagation
  - for systems that can be described through a LINEAR model
  - in which system and measurement noises are WHITE and GAUSSIAN
- Under these assumptions,
  - the conditional pdf is Gaussian
  - mean=mode=median
  - there is a unique best estimate of the state
  - the KF is the best filter among all the possible filter types.

Meaning of the covariance matrix

Generals on Gaussian pdf

- Let z be a Gaussian random vector of dimension n

$$E[z] = m, E[(z-m)(z-m)^T] = P$$

- P - covariance matrix - symetric, positive defined
- Probability density function

$$p(z) = \frac{1}{\sqrt{2\pi^n detP}} exp[-\frac{1}{2}(z-m)^T P^{-1}(z-m)]$$

Thank you

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