PRICING OF ENERGY BY MEANS OF STOCHASTIC MODEL

by

Emmanuel Evarest

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CERTIFICATION

The undersigned certify that they have read and hereby recommend for acceptance by the University of Dar es Salaam the dissertation entitled: **Pricing of Energy by means of Stochastic Model**, in partial fulfillment of the requirements for the degree of Master of Science (Mathematics) of the University of Dar es Salaam.

Prof. M. Heilio

(First Supervisor)

Date:..................................

Dr. W.C. Mahera

(Second Supervisor)

Date:...................................

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I, **Emmanuel Evarest**, declare that this dissertation is my own original work and has not been presented and will not be presented to any other university for a similar or any other degree award.

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ABSTRACT

In this dissertation we present a mean-reverting jump diffusion model for pricing of energy particularly electricity by means of stochastic model. We discuss the stochastic model which is used to model the behaviour of electricity prices. Despite some distributional similarities with asset prices, electricity prices have dramatically different stochastic properties from those of financial products and even other commodities due to its non-storability nature. These properties include mean-reversion, stochastic volatility, seasonality as well as short lived spikes or jumps. The recent deregulation of electricity markets in the world has exposed power producers and users to market risk due to these unique features of energy price dynamics. The prices contain strong mean reversion, which reflects the demand and supply movements. The model developed is calibrated using the market data from Nordpool for the period from January 1997 to April 2000. The daily price model is estimated via Maximum Likelihood-Conditional Characteristic Function (ML-CCF) to obtain the solution in closed form. Then we simulate the model from the parameters obtained, we found that the simulated and real price series have similar trends and covers the same price ranges. From the model we derive the corresponding forward prices under Q-martingale measure and calculate forward prices at different expiries. All forward prices are subject to the market price of risk due to the fact that power markets are incomplete markets. The ability to model the spot prices and obtain forward price dynamics is essential when assessing the performance of headging strategies that use forward contracts.

Contents

List of Tables

List of Figures

CHAPTER 1

INTRODUCTION

1.1 General Introduction

The energy industry is fundamental to the natural economy and is paramount to socio-economic development and the improvement of people's living standards. In the environment of a rapidly expanding demand, energy pricing is confronted with dual pressures for economic status and environmental protection. Energy commodity markets grow rapidly as the restructuring of electricity supply industry is spreading around the world. The global trend of energy market reforms inevitably exposes the portfolios of generating assets and various supply contracts held by traditional electric power utility companies to market price risks. It has so far changed and will continue to change not only the way a utility company operates and manages its physical assets such as power plants, but also the way a utility company values and select a potential investment projects.

One of the key aspects towards a competitive market is deregulation, where price controls are removed and thus encouraging competition. That is, energy prices are no longer controlled by regulators and now are essentially determined according to the economic rule of supply and demand. At the same time, the deregulation and restructuring of electricity markets has paved the way for a considerable amount of trading activity, which provides utilities with new opportunities but more competition as well. Price variations of energy have increased significantly as a consequence of the introduction of competition, encouraging the pricing of a new breed of energy-based financial products to hedge the inherent risk, both physical and financial, in this market. Most of the current transactions of instruments in the energy markets are carried out through bilateral contracts ahead of time, although it can also be traded on forward and futures markets and through power exchanges. The most striking difference that singles out energy markets particularly electricity markets is that electricity is very difficult or too expensive to store,hence markets must be kept in balance on a second-by-second basis [10]. This can be done by the National Grid Company, which operates the balancing mechanism to ensure system security.The consequence of the deregulation is that the energy prices are determined by the coactions between demand (the agents who buy energy and then sell it to the consumers) and supply(generators) in what is known as a "pool". The result is that the suppliers compete in selling electricity in the pool while the agents purchase it from the market pool at prices of equilibrium that are set at a point of intersection of supply and aggregated demand [33]. Thus, causing the volatility of the new deregulated prices to be extremely high.

In general, in a deregulated energy market, asset valuation and their related risk management requires in-depth understanding and sophisticated modeling of commodity spot prices [17]. In this study, we investigate modeling of energy commodity prices in the cases where the underlying commodities may be very costly to store. Among all the energy commodities, electricity poses the biggest challenge to model its price behaviors, because electricity can not be stored or inventoried economically once generated. So the use of forward and 'spot' is crucial. The spot market for electricity is an auction, where generators and distributors submit hourly/daily supply and demands curves, and the equilibrium price is the resulting market price. In the near future, especially due to the plans of introducing power grids between the countries for energy business and the private sector which has just recently been allowed to generate power and sell in the country, will obviously introduce suppliers who would purchase from various sources and sell. Introduction of energy markets in Tanzania and East Africa region in general will promote efficient gains from higher efficiency in the operation of generation, transmission and distribution services; stimulate technical innovations that will lead to efficient investment, and clear trends of falling of electricity prices and more efficient use of assets in energy sector will be visible. Therefore understanding and characterizing of the structure of the energy prices is essential in the deregulated energy market for sustainable development.

1.2 Definition of Terminologies in the Theory of Pricing.

1.2.1 The commodity market

Energy commodity markets comprises of energy production, energy distribution business and energy trade. Both the input (with the exception of hydro plants) and the output products of the energy business are traded on well established markets and exchanges. Generally speaking, the corresponding commodities (coal, oil, gas, etc. as inputs, electricity as output) posses some particularities that prevent us from using standard financial theory in pricing their derivatives such as options, forwards, and futures (which are the most widely used products for risk mitigation). Energy commodity market can be complete or incomplete depending on the hedgeability of contingent claims in the market.

1.2.2 Complete market

A market is complete with respect to a trading strategy if there exist a self-financing trading strategy such that at any time t, the returns of the two strategies are equal. In general a complete market is a market in which every contingent claim can be replicated by trading in the underlying asset or assets. That means a market must be possible to instantaneously enter into any position regarding any future state of the market. Also a market is dynamically complete if it is possible to construct a self-financing trading strategy that will have the same cash-flow. In other words, a complete market allows you to place your entire bet at once, while a dynamically complete market may require that you execute subsequent trades after making your initial investment. The requirement that the strategy be self-financing means that subsequent trades must be cash-flow neutral (you cannot contribute or withdraw any additional funds). Any complete market is also dynamically complete.

1.2.3 Incomplete Market.

At any given time at the stock market, the stock price can increase or decrease slightly or fall a lot. It is not possible to hedge against all these increase or decrease in price simultaneously because there is no opportunity to carry out a continuous-changing delta hedge, this leads to impossibility of perfect hedging [26]. The impossibility of perfect hedging means that the market is incomplete, that is not every option can be replicated by a self-financing portfolio.

1.2.4 Energy derivatives.

The emergence of the energy markets has given birth to energy derivative markets. An energy derivative is a financial contract whose value depends on energy price. For example, a forward contract is an obligation to buy or sell electricity for a predetermined price at a predetermined future time [22]. By definition, a derivative security (an option being part of this larger family) is a security whose price depends on or is derived from one or more underlying assets. The derivative itself is a contract between two or more parties. Its value is determined by the price fluctuations of the underlying asset. The most common underlying assets include: stocks, bonds, commodities, currencies, interest rates and market indexes. Two of the most widely used such derivative securities (also called contingent claims) are the forward contracts and the futures contracts. A forward is a contract in which delivery of the underlying commodity is referred at a later date than when the contract is written with the price of delivery being set at the time of contracting. In futures contract, the settlement of the net value is started immediately after making the contract, and it is carried out daily until the end of the delivery time.

1.2.5 Options

Options are financial instruments that convey the right, but not the obligation, to engage in a future transaction on some underlying security. An option contract binds only the seller of the option. The buyer of the option is required to pay a premium to the seller as a compensation for the risk taken by the seller. There are two types of options call and put options; For example, buying a call option provides the right to buy a specified amount of a security at a set strike price at some time on or before expiration, while buying a put option provides the right to sell [7]. Upon the option holder's choice to exercise the option, the party who sold, or wrote, the option must fulfill the terms of the contract. Rights and obligations related to option contracts are illustrated in Figure 1.1.

1.2.6 Electricity trade.

The reform of the electricity market in different countries in the world has removed obstacles to competition in the sectors of the market where competition is possible, that is, generation and sales. Now, it became possible for the end-users of electricity to invite tenders from electricity suppliers [31]. Previously, the supplier of electricity had automatically been the local electricity company operating in the area; now the market reform brought new, versatile alternatives to purchasing of electricity also for large-scale consumers and retailers. In the old regime, the possible acquisition methods were long-term delivery contracts made with electricity producers, or the ownership of power plants or

	Buyer	Seller
Call options	A right to purchase	An obligation to sell
Put options	A right to sell	An obligation to purchase

Figure 1.1: Rights and obligations related to option contracts.

power plant shares. In the new market situation, a new supply channel is created alongside bilateral contracts as it became possible to trade electricity at the power exchange. This is what is taking place in the countries where electricity markets have already introduced. For example Scandnavian countries at Nordpool.

1.2.7 Power exchange.

A power exchange is an open, centralized, and neutral market place, where the market price of electricity is determined by demand and supply. A high liquidity ensures that the market price at the power exchange is a “correct” price [40]. The products sold at the exchange are standard products, and the communication is equitable to all actors on the market. The counterparty in trading at the power exchange is always the power exchange, trading being thus anonymous, and the risk of counterparty failure is avoided. The operation of the power exchange is market-oriented, in other words, the members of the power exchange participate in decision making. Thus, it is possible to make the product structure of the power exchange meet the needs of the market parties.

1.2.8 Over The Counter (OTC) Markets.

OTC markets refer to all wholesale trade in electricity outside power exchange. Also the traditional electricity wholesale based on bilateral contracts is a part of the present OTC markets. With the services provided by the OTC markets, it is possible for the actors on the market to tailor their portfolios of purchase and sale contracts to accurately meet their needs. Unlike in the trading at the power exchange, there is a risk of a counterparty default. The power exchange and the OTC markets that complement each other together form a well-functioning market mechanism for the wholesale of electricity, the objective of which is to control the high volatility of the electricity market prices.

1.2.9 Stochastic process.

A stochastic process is a collection of random variables $\{X(t), t \in T\}$, where T is the index set of the process and t denotes time. $X(t)$ is the state of the process at time t. The set of possible values that $X(t)$ can take is called the state space of the process and is denoted by S. The stochastic process is classified as discrete time when the index set is finite or countable infinity, ,i.e, $T = \{X(t), t = 0, \pm 1, \pm 2, ...\}$ or $\{X(t), t = 0, 1, 2, ...\}$ e.g. time in months, years etc. Otherwise it is continuous where T is defined as $T = \{X(t) : -\infty < t < \infty\}$ or $T = \{X(t) : t \geq 0\}$.

1.3 Stylized features of Electricity Spot Prices.

1.3.1 Why do Electricity Prices Exhibit so High Volatility?

Several elements can explain the high volatility of the electricity prices. However, the most important one is the non-storability of the commodity. Electricity is not possible to be stored physically (only indirect storage can be realized through hydroelectric plants or storage of generator fuel). Consumption and production have to be balanced in a continuous process, and as a result, shocks in demand and supply have a direct influence on the equilibrium prices since they cannot be smoothed out easily. The general features of demand and supply play an important role as well in the high volatility that is observed. Demand and supply's intersection is what determines Pool's prices. If the levels of demand are relatively low then suppliers use base-load units with quite low marginal costs, and as soon as larger quantities are needed then new generators get into the system with much more high marginal costs. Hence, the curves of demand and supply during peak times are very steep, and we can observe the prices to increase sharply as the quantity of demand increases.

1.3.2 Seasonality.

The major factors that explain the seasonality of electricity prices are business activities and weather conditions. There are various seasonality patterns that one can find in data such as intra-daily, weakly as well as monthly seasonality. It is well known that electricity demand exhibits seasonal fluctuations. They mostly arise due to changing climate conditions, like temperature and the number of daylight hours. In some countries also the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which varies from season to season. These seasonal fluctuations in demand and supply translate into the seasonal behavior of spot electricity prices.

1.3.3 Volatility.

Another feature of electricity prices is the evidence of extremely high volatility. The volatility encountered in electricity markets is exceptional and not comparable with the one observed in other commodity and financial markets. Applying the standard concept of volatility, the standard deviation of the returns on a daily scale, Weron [48] obtains:

- notes and treasury bills less than: 0.5%
- stock indices: $1 - 1.5\%$
- commodities like natural gas or crude oil: $1.5 - 4\%$
- very volatile stocks: not more than 4%
- and electricity up to 50%

The high volatility pattern is due to the transmission and storage problems and of course the requirement of the market to set equilibrium prices in real time. It is not easy to correct provisional imbalances of supply and demand in the short-term. Therefore, the price changes are more extreme in the electricity markets than other financial or commodity ones.

1.3.4 Mean-Reversion.

Mean-Reversion is the tendency of a stochastic process to return over time to a long-run average value. Energy spot prices are in general regarded to be mean reverting or anti-persistent, and the speed of mean reversion depends on several factors, including the commodity itself being analyzed and the delivery provisions associated with the commodity. When the price of a commodity is high, its supply tends to increase thus putting a downward pressure on the price; when the spot price is low, the supply of the commodity tends to decrease thus providing an upward lift to the price. Thus, in a long run prices will move towards the level dictated by the cost of production.

1.3.5 Jumps and Spikes.

Jumps or spikes are the abrupt and unanticipated extreme changes in the spot prices. Within a very short period of time, the system price can increase substantially and then drop back to the previous level. Jumps in the spot prices are an effect of extreme load fluctuations, caused by severe weather conditions often in combination with generation outages or transmission failures. These spikes are normally quite short lived, and as soon as the weather phenomenon or outage is over, prices fall back to a normal level [48]. The spiky nature of spot prices is the effect of non-storability of electric energy, ,i.e, electricity to be delivered at a specific hour cannot be substituted for electricity available shortly after or before. Other factors include the production stack and the demand curve, and how the spot price depends on those two. Some of power generations are more expensive than others. If a larger fraction of power needed to satisfy the demand comes from expensive sources, price will go up (see Figure1.4). Also the bidding strategy used by some of the potential buyers can cause spikes, because they place bids, on regular basis at a maximum allowed level.

1.4 How the energy spot price is determined?

The energy spot price is the result of two-sided uniform price auction for hourly or half an hour time intervals depending on arrangement of the administration of the particular energy market (see Figure 1.2). It is determined from various bids presented up to the time when the auction is closed. The system price is determined by the market equilibrium ,i.e, the point where supply and demand curves cross (see Figure 1.3).

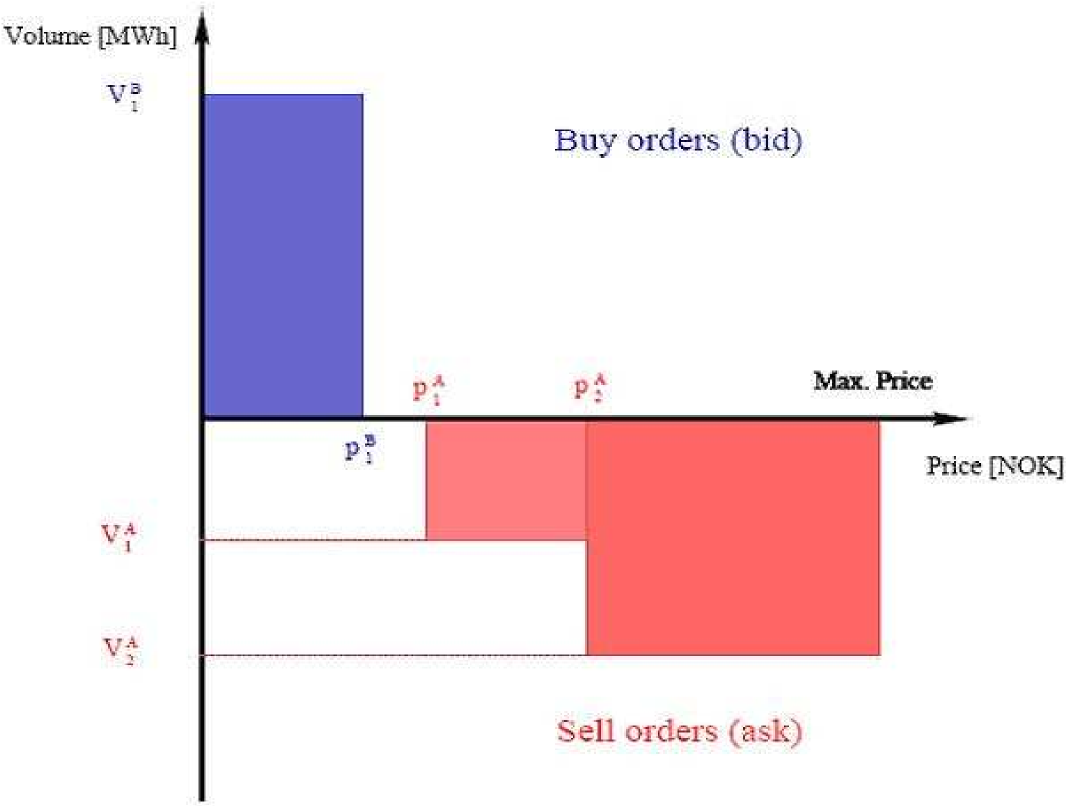

Figure 1.2: The (bid and ask) orders for a given hour of a fictitious power generator. At energy spot market buy orders are positive while those of sell orders are negative. In this particular example there is one purchase order of V_1^B MWh at a maximum price of P_1^B , and two sell orders. The two sell orders (asks) are for volumes V_1^A and V_2^A MWh and the sell prices are set to atleast P_1^A and P_2^A respectively.

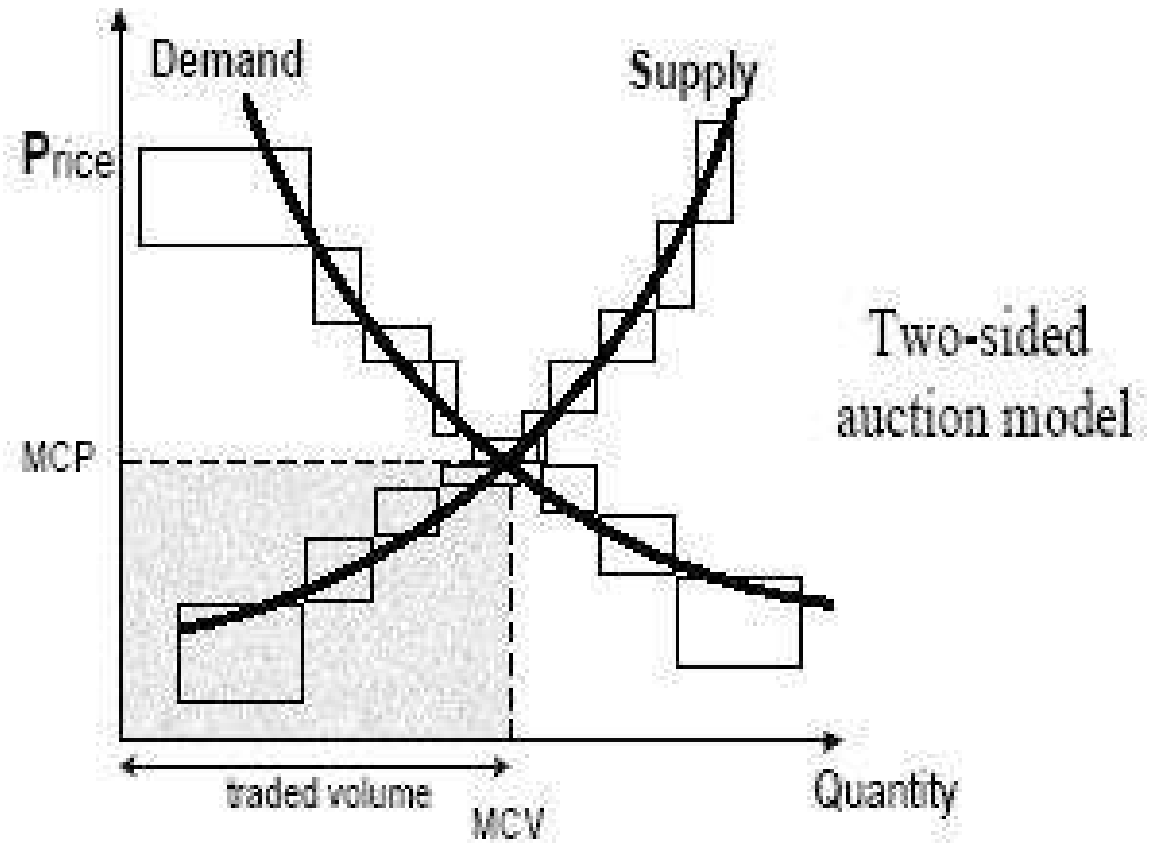

Figure 1.3: Two sided auction showing MCP.

MCP is the intersection of the supply curve(constructed from aggregated supply bids) and demand curve (constructed from aggregated demand bids).

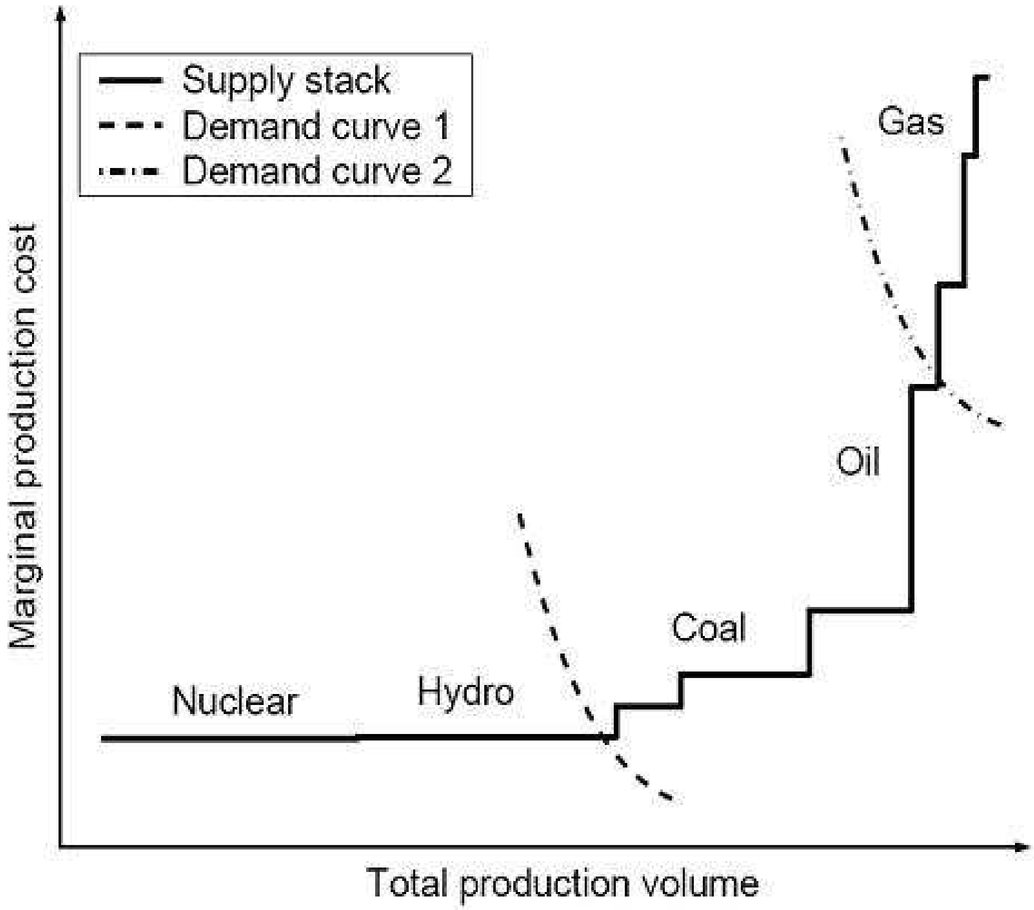

Figure 1.4: A schematic supply stack with two potential demand curves super imposed on it .

The spot price given as the intersection of supply and demand is not very sensitive to demand shift when demand is low (curve1), since the supply stack is typically flat in low demand region. However when demand is high, small increase in demand can have huge effect on price (curve2).

1.5 Background of the TANESCO Limited.

TANESCO is a parastatal organisation established in 1964 and in 1968; the company changed its name to the Tanzania Electric Supply Company Limited, till now. It is wholly owned by the government of Tanzania and is under the Ministry of Energy and Minerals. The company's core business is to generate, transmit, distribute and sell electricity to customers in Tanzania mainland and bulk supply to the island of Zanzibar. TANESCO's generation system consists of hydro, thermal and gas, where hydro contributes the lion's share of TANESCO's power generation. The company's total installed capacity is 793MW of which 561MW is hydropower and the rest 232MW is thermal and gas. About 96% of the installed capacity is located in the interconnected grid. The rest are isolated diesel power stations.

Several major power development and construction projects have therefore been undertaken during the past three and half decades, including construction of new hydropower stations, transmission and distribution networks. Construction of the national grid system; numerous 220 kV, 132 kV, 66 kV, 33 kV and 11 kV transmission distribution lines as well as 400V / 230V lines connecting customers. Many isolated diesel power stations and rural electrification schemes have been constructed. However, about only 11 percent of the countrys estimated population of 34 million have access to reliable electricity.

The existing interconnected grid network consists of 2,781km of 220kV, 1,403km of 132kV, and 459km of 66kV. Distribution network consists of 8,325km of 33kV, 3,732km of 11kV and 12,992km of lower voltages. In 2002, there were about 473,546 domestic, light commercial and light industrial customers and 1,335 large industrial and commercial customers lined by these distribution lines.

Two private independent power projects (IPP's) which are connected to TANESCO grid are IPTL (Independent Power Tanzania Ltd) with 100 MW installed capacity and SONGAS (Songo Songo gas to electricity project) which by the end of 2005 had 200 MW capacity although more gas turbines will be installed to increase the capacity. TANESCO also imports 10 MW of electric power for Kagera Region from Masaka substation in Uganda while Sumbawanga, Tunduma and Mbozi districts receive about 3 MW from neighbouring Zambia. Bulk supply of electricity is made to Zanzibar from Ras Kilomoni substation at the Indian Ocean coast in Dar es Salaam [45].

1.5.1 Metering customers.

The metering system for TANESCO customers are of two types. Pre-paid Metering System (LUKU): LUKU is a Swahili abbreviation for "LIPA UMEME KADIRI UNAVYOTUMIA" which means "pay for electricity as you use it". The modern LUKU meters are becoming extremely popular because customers can purchase electricity like any other product such as matchboxes or candles; customers can plan and budget for electricity usage; they can control the amount of electricity they use; can embark on voluntary conservation, and can purchase their electricity at any convenient sales point. The software has been designed mainly for residential and commercial consumers, not industrial type. The LUKU system which was established in 1995 has improved customer services by offering the customer the above mentioned advantages. For example, there are 32 LUKU vending stations in Dar es Salaam from where LUKU users can purchase their electricity. The second type is Credit meters (conventional meters): These meters allow for customers to be billed after consuming electricity for a month.

TANESCO uses different tariff rates to different customers groups depending on the quantity of electricity consumed. These include domestic, small business and industrial consumers. In East Africa, TANESCO offers the lowest tariff rates for domestic and small business consumers, while Uganda Electricity Board (UEB) charges the lowest tariff rates for industrial customers followed by TANESCO and KPLC (Kenya Power and Lighting Company) as shown in Figure 1.5 to 1.7.

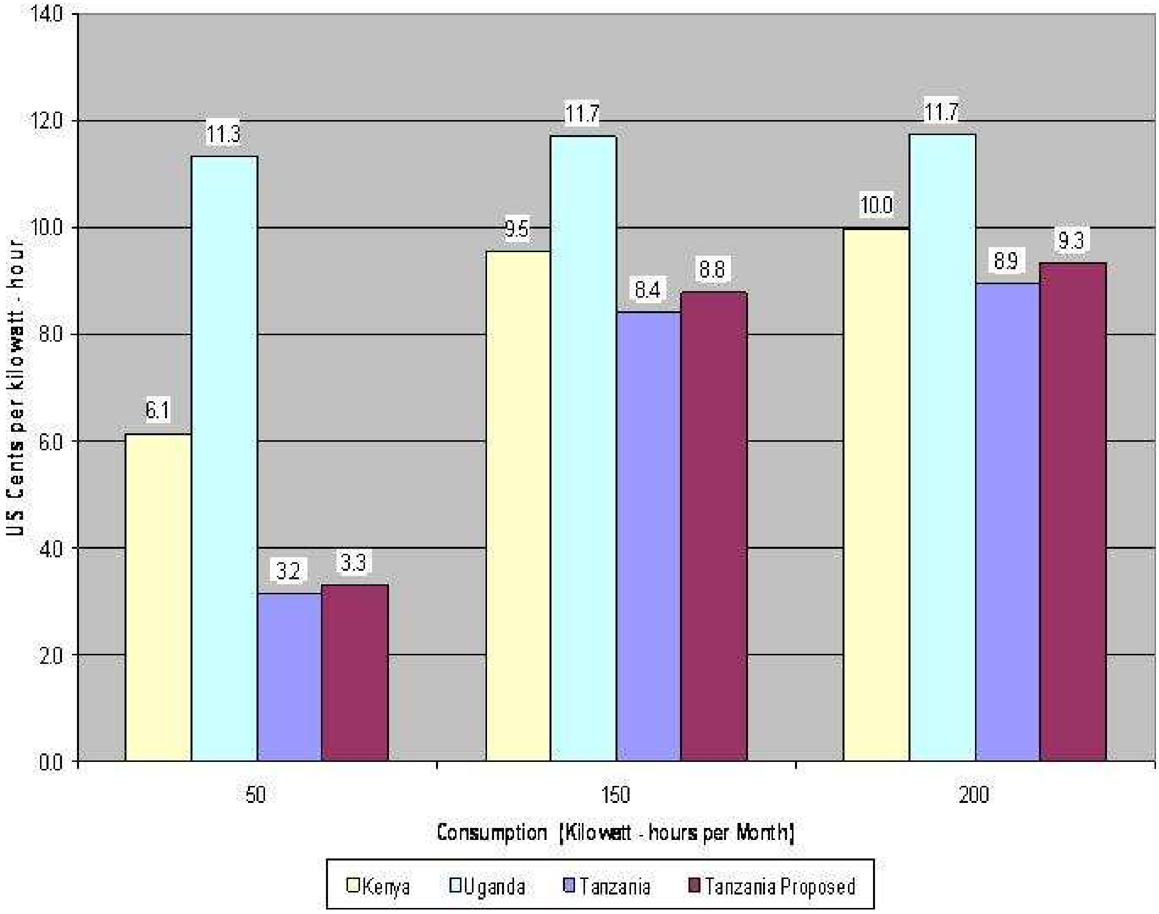

Figure 1.5: Tariff comparison for Domestic consumers in East Africa.

1.5.2 Inter connections with neighbours.

TANESCO actively cooperates with various Governments and other Power Utility bodies, in order to enhance power development in the country. The major areas of cooperation include Southern African Power Pool (SAPP), Nile Basin Regional Power Trade Project and Nile Equatorial Lakes- Subsidiary Action Program (NELSAP). SAPP was created in

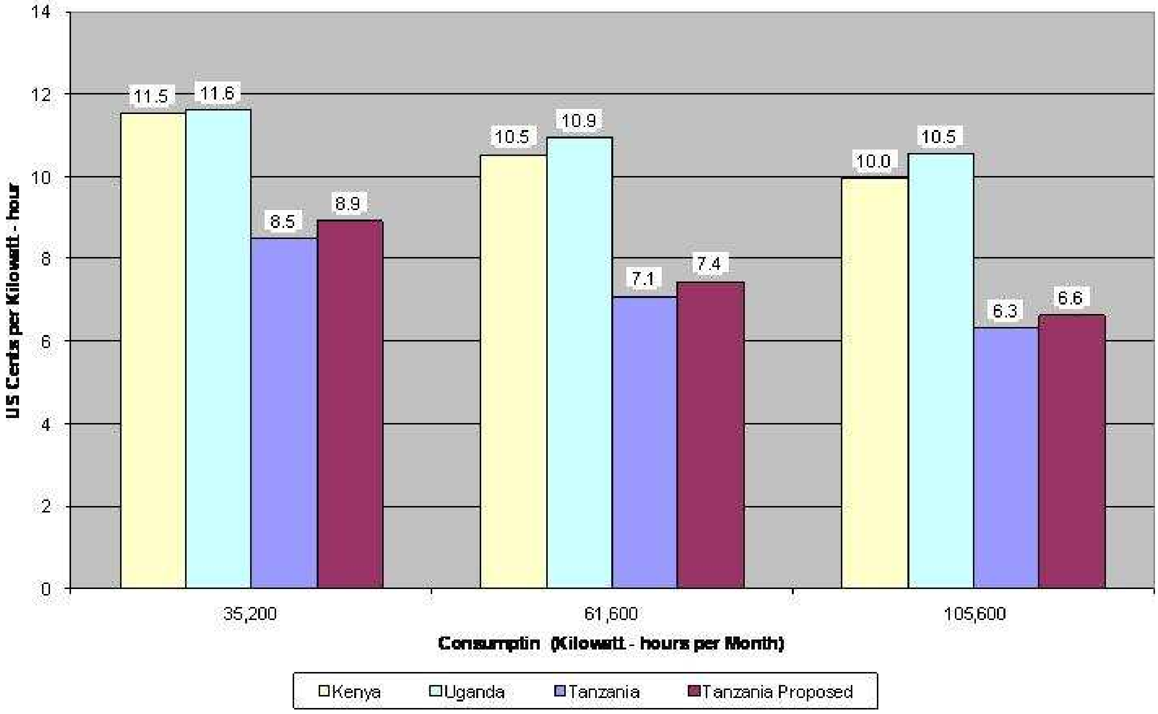

Figure 1.6: Tariff comparisons for small business consumers in East Africa.

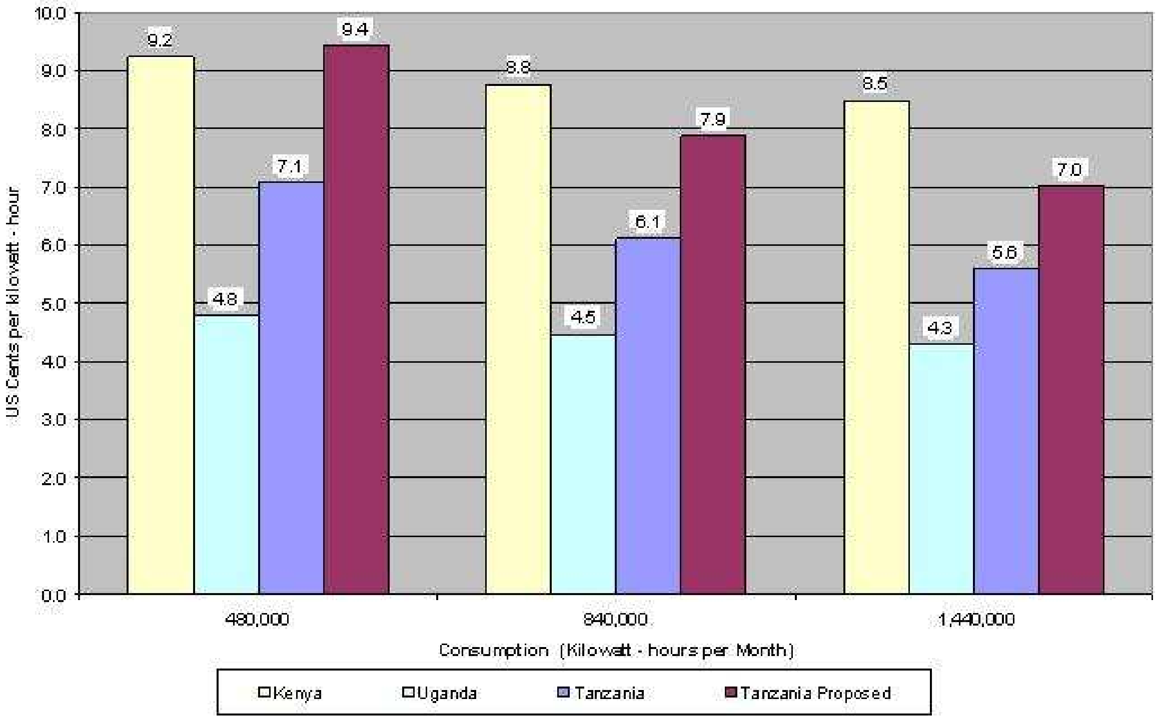

Figure 1.7: Tariff comparisons for industrial consumers in East Africa.

SADC countries as an effort to pool their electricity-supply resources for their mutual benefit. SAPP project include Zambia-Tanzania-Kenya interconnection, Malawi-Mozambique interconnection and DRC-Zambia interconnection. It is envisaged that transmission lines will be constructed between Serenje (Zambia), Mbeya (Tanzania), Arusha (Tanzania) and Nairobi (Kenya), and the capacity of the links may be increased to 400 MW from an originally-envisaged 200 MW. This connection should reduce the cost of power through out the Great Lakes region [49]. Tanzania being part of NELSAP enables the country to benefit from NEL projects consisting of small scale hydropower development in critical areas, and strengthening transmission interconnections between several countries in the NELSAP region (Burundi, DRC, Kenya, Rwanda, Tanzania and Uganda). Also Kenya and Tanzania are discussing on a power supply agreement under which Tanzania's Electricity Supply Company (Tanesco) will supply power to the Kenyan border towns of Lunga Lunga and Vanga. To increase the country's access to additional power supply, Kenya in the same month joined in the memorandum of understanding to set up the so-called Eastern Africa Power Pool of regional countries working toward faster electrification, increased cost-effectiveness and quality of reliable supply in the region [28]. Also Tanzania has an added advantage that is ideally situated to do cross border trading with her neighbours that will enable it to maximize the potentials offered by the countries especially in electricity consumption and marketing pattern.

1.6 Statement of the Problem.

Prices are very important so long as they stabilize and distribute incomes, control buying power and harmonize production and consumption of commodities. They also influence producers to supply at appropriate level and thus regulating demand for goods and services in the market. Therefore energy pricing techniques are of great importance in insuring marketing efficiency in the case variations of costs of energy supply at the energy market.

Every customer/consumer is interested only in the amount of money that she/he will spend on her/his electricity consumption. This money amount is a stochastic variable that depends on the electricity price and the amount of consumption at each moment of time. Because different customers have different consumption behaviours, the dynamics of the money amounts are different. Thus, pricing techniques are of great importance to achieve fair price for both demand and supply side.

In this study we presented the mathematical tools in finance that are applicable to the problem of pricing in energy trade. The stochastic nature of the process is taken into account and appropriate reference to Tanzanian energy system is highlighted. The available time series data from the history of consumption and production from foreign energy markets especially Nordpool have been analyzed to illustrate the methods.

1.7 Research Objectives

1.7.1 General Objectives.

The main objective of this research is to develop energy pricing model and examine how stochastic modeling is applied in pricing of energy in varying conditions of supply and demand of electric energy.

1.7.2 Specific Objectives.

Assuming that the market is efficient, the specific objectives are:-

i. To determine the fair price of energy option and derivatives for both writer (holder) and buyer at a particular energy exchange market.

ii. To determine the accuracy of the mean-reversion jump-diffusion model with time series data of daily energy prices at the market.

1.8 Significance of the Study

The research intends to study pricing techniques of energy particularly electric energy by means of stochastic models. Additional interest is to study the general behaviors of energy markets pricing models and find the solution of problems of fair price of energy options and derivatives in the market. Market based energy pricing is not yet taking place in Tanzania as still there is vertical integration of electrical energy, however it is bound to take place in the near future. Future trends includes the envisaged East African regional power plan which aims to set an Eastern Africa Power Pool (EAPP), whose objective is to set a framework for power exchanges between utilities of the member states. TANESCO also participates fully in the East Africa Community Energy Committee whose major objective is to prepare the East Africa Power Master Plan and the Energy Committee for the proposed Zambia Tanzania- Kenya interconnection. Also the Southern African Power Pool (SAPP) in the SADC countries has expansive projects; some of the SAPP projects include Zambia-Tanzania-Kenya interconnection [45]. Therefore this research is useful in the sense that, the energy pricing techniques will be helpful to power companies in the country to run the exercise of pricing of energy particularly electricity when it will be effectively taking place.

CHAPTER 2

LITERATURE REVIEW

Several researches have been conducted on energy option pricing using varieties of models.

Schwartz *et al*, (1997) investigated several stochastic models for commodity spot prices and performed an empirical analysis based on copper, gold and crude oil price data. They found that stochastic convenience yields could explain the term structure of forward prices and demonstrate the implications to hedging and real asset valuation by different models. Cartea and Figueroa (2005) pointed out that, in stock markets, prices are allowed to evolve 'freely', but this is not true for electricity prices; these will generally gravitate around the cost of production. Under abnormal market conditions, price spreads are observed in the short run, but in the long run supply will be adjusted and prices will move towards the level dictated by the cost of production. This adjustment can be captured by mean-reverting processes, which in turn may be combined with jumps to account for the observed spikes. Lucia and Schwartz (2002) propose and estimate a one and two-factor mean reverting models with deterministic seasonality for the Scandinavian market (NordPool), showing that the seasonal pattern in spot electricity prices could explain part of the shape of the observed term structure of futures prices. Bhanot (2000) analyses electric power prices using data from 12 regional markets from the US focusing in the mean-reverting and seasonal behavior of the series and on the possible regional differences among them.

Ethier and Mount (1998) applied regime-switching models to electricity prices, they proposed a two state specification in which both regimes were governed by AR(1) price processes with common or different variances. According to their studies of on-peak prices from SERC, ECAR and PJM they concluded that there were strong support for the existence of different means and variances in the two regimes. Huisman R. *et al* (2003), proposed a regime switching model that models price spikes separated from normal mean-reverting prices, that is, a mean reverting process that is not directly associated with jumps. In the regime switching model, they assumed that electricity price is in one out of the three different regimes at each point in time. They identified a normal regime that can contain a mean reverting component. In addition they identified two extra regimes: the first regime models a price jump and a second regime models the way the process falls back to the normal process. Markov transition matrices specify the probabilities that the electricity prices move from one regime to another from one time point to the next. That means;

- the base regime $R_t = 1$ modeling the 'normal' electricity price dynamics,
- the initial jump regime $R_t = 2$ for a sudden increase (or decrease) in price,
- the jump reversal regime $R_t = 3$ that describes how prices move back to the normal regime after the initial jump has occurred.

Weron *et al* (2004) extended the model by allowing log-normal and Pareto distributed spikes regimes the regime switching models produce significantly more spikes than could be observed in real data.

Knittel *et al* (2001), presented an empirical analysis of deregulated electricity prices, where they revealed that, conditionally Gaussian models have little chance of accurately representing the data generating process because of their inability to capture very large changes in prices. Heavy-tailed random components, such as Student's-t and Levy processes, are likely candidates for future specifications. Second, univariate Markovian specifications are unable to accommodate the persistence found in prices. Depending on the frequency of one's data, higher order lags are needed to capture the autocorrelation in the price series. Finally, exogenous information, such as marginal costs, is likely to be another important addition to future modeling efforts. Routledge *et al.* (2001) develop a competitive rational expectations framework for non-storable electricity and storable potential fuels. Various empirical features followed directly from their equilibrium analysis, including skewed spot price distributions, price dependent heteroscedasticity with higher (lower) volatility when prices are higher (lower), and unstable electricity-fuel correlations. They showed that skewness and heteroscedasticity were immediate consequences of inelastic demand, combined with a possibly discontinuous supply curve (the "supply stack"). The supply curve was flat at low outputs, corresponding to inflexible base load generation using efficient and cheaper power sources whose output is expensive and time-consuming to regulate, such as nuclear energy, but becomes very steep at high outputs, corresponding to flexible peak generation using costly and less efficient standby power sources including gas-fired power plants.

Keppo and Rasanen (1999) analyzed the problem of pricing of electricity tariffs in energy open markets with the assumption that both the customers' power consumption and the market prices are stochastic processes. They showed that the more there is uncertainty about the customers' consumption, the higher the fixed charge of the price tariff contract should be. In his pricing model, every customer is interested only in the amount of money that she/he will spend on her/his electricity consumption. This money amount is a stochastic variable that depends on the electricity price and the amount of consumption at each moment of time. Because different customers have different consumption behaviours, the dynamics of the money amounts are different. They assume that each customer will consume electricity in future according to a given stochastic consumption model and therefore, they price the energy options (contracts) based on that amount of money (customers' willing to pay for electricity).

Lari-lavassani *et al* (2001), discussed mean-reversion process for energy prices that includes one to three factor models for energy pricing. They pointed out the major problem in the practical use of these multifactor models is that direct market data for analyzing those models only exists for the spot price S_t of the commodity, where as these models have additional hidden variables such as the long run mean L_t or the stochastic volatility σ_t . They proposed two basic alternatives options, one is to model the forward curve directly and possibly build an implied model of the underlying spot price as a second step. The other option is to model the spot price and to deduce from this a model for the forward curve. Bessembinder and Lemmon (2001) proposed a method in which forward prices are found using the condition that they provide equilibrium in demand for forward contracts. In the paper by Sapatgiat *et al* (2001) market clearing prices are determined by solving a Nash equilibrium problem for the bidding strategies of market participants. Weron (2006) comments that statistical models are potential candidates for modeling electricity spot price dynamics, because they capture the seasonality prevailing in electricity price processes during normal non-spiky periods. And quantitative (or stochastic) models of price dynamics are required to accurately recover the main characteristics of electricity prices, typically at the daily time scale.

Hinz (2004), described that, the price and the quantity of electricity produced and consumed are determined by generators costs and by consumers' willingness to pay. He explained that in practice, two technical problems prevent electricity to be traded as usual commodity.

- The first problem, *the balancing problem*, is that the demand and the supply of electricity has to be equal at any time.
- The second problem, *the constraints problem*, is that the trading is restricted by congestion of the electrical grid.

To ensure the imbalance of energy production and consumption, the balancing electricity trading is effected within an auctionlike environment, whereas contracts on future electricity delivery are traded conveniently. Thus, a deregulated electricity market consists at least of two parts: the balancing market for contracts on the immediate electricity production and the electricity exchange for those on the future delivery. Within the last segment, a remarkable trading activity is observed for hourly contracts with delivery within the following day, at the socalled spotmarket. The market for other forward a contract is referred to as futures market. The structure of deregulated electricity market is shown in Figure 2.1.

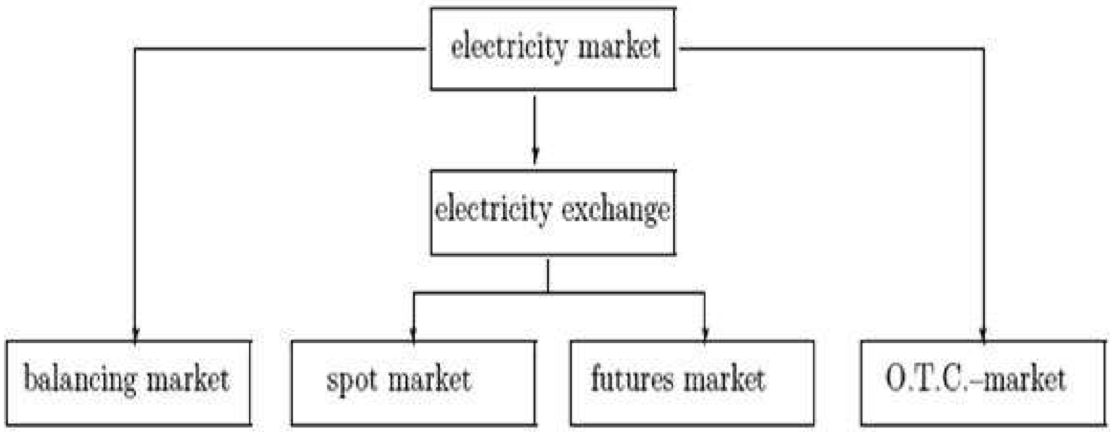

Figure 2.1: Structure of deregulated electricity market.

Mavrou (2006), described the stylized facts of electricity prices in which they gave reasons for volatility of electricity prices and its seasonality the most important one is the non-storability of the commodity. ,i.e., Electricity is not possible to be stored physically (only indirect storage can be realized through hydroelectric plants or generator fuel). He further identified that, the general features of demand and supply play an important role as well as in the high volatility that is observed. The demand and supplys intersection is what determines pools prices. So demand being relatively insensitive, combined with constraints which supply can encounter during peak time's lead to the fact that short term energy prices are highly volatile. Hence, the curves of demand and supply are very steep, and we can observe the prices to increase sharply as the quantity of demand increases. In addition, according to the structure of the market and the generator's market, only few suppliers could be sufficient enough to satisfy the remaining demand. Thus the behavior of the generators becomes monopolistic or oligopolistic.

About seasonality, he explained that, the major factors that explain the seasonality of electricity prices are business activities and weather conditions. There are various seasonality patterns that one can find in data such as intra-daily, weakly as well as monthly seasonality. He assumed that the factors that generate the seasonality are deterministic and since the daily average system prices are used, only monthly and weakly patterns are considered. He further explained that, high volatility pattern is due to the transmission and storage problems and of course the requirement of the market to set equilibrium prices in real time. It is not easy to correct provisional imbalances of supply and demand in the short-term. Therefore, the price changes are more extreme in the electricity markets than other financial or commodity ones.

Benth (2003), explained the completeness and incompleteness of markets, that buying electricity in a complete market on the sport market is far from being the same as buying shares in some company quoted on a stock exchange. Electricity is a physical product, unlike a stock in which most exchanges is simply virtual paper existing in some computer; it must be used once you have bought it. You can not of course store electricity and therefore it is not suitable for hedging of an option. However if electricity is produced by hydro power, you can store it indirectly in a reservoir, but this is very expensive hedging approach.

Another aspect he has touched is incompleteness due to stock price dynamics which deviates from geometric Brownian motion. For example, if the stock price is modeled by a geometric NIG Levy process or by some stochastic volatility model, the market becomes incomplete. The degree of incompleteness increases even more if we add transaction costs and constraints on the hedging positions. The energy market is incomplete because we cannot use the spot for hedging. Girsanov theorem which states that, For $\lambda \in \mathbb{R}$ define the probability

$$Q(A) := E[I_A M(T)],\ A \subset \Omega,$$

where

$$M(T) = \exp(-\lambda B(T) - \tfrac{1}{2}\lambda^2 T).$$

This theorem produces probabilities Q which are equivalent to P and for which a certain process becomes a Brownian motion. Then from the theory, he found the arbitrage free prices for a call option on the spot electricity with strike K at exercise time T is given by

$$P(0) = \exp(-rT)E_Q[max(S(T) - K, 0)]$$

where Q is an equivalent martingale measure.

Gonzalez *et al* (2005) proposed an Input-Output Hidden Markov Model (IOHMM) for analyzing and forecasting electricity spot prices. The model provides both good predictions in terms of accuracy as well as dynamic information about the market, where different market states were identified and characterized by their more relevant explanatory variables. Moreover, a conditional probability transition matrix governs the probabilities of remaining in the same state, or changing to another, whenever a new market session is opened. He concluded that, the market evolves along time through different "market states", which are mainly characterized by the interaction among resources, demand, and participants strategies. This sequence of "market states" is reflected in the electricity prices time series as different regimes in the dynamics of the prices. Pao (2006) proposed a new artificial neural network (ANN) with single output node structure by using direct forecasting approach, where he investigated the ANNs by employing a rolling cross validation scheme. Out of the sample performance evaluated with three criteria across five forecasting horizons showed that the proposed ANNs were more robust multi-step ahead forecasting method than autoregressive error models.

CHAPTER 3

STOCHASTIC MODEL FOR ENERGY PRICES.

3.1 Introduction.

The mathematical model developed in this chapter is used to describe the pricing techniques of energy particularly electricity in a deregulated electricity commodity market. Through the observed variation of prices we establish the cause of the movement of prices for a given period of time. So a mean-reverting jump-diffusion model is developed and analyzed, and then we exploit the transform analysis introduced by Duffie *et al* (2000) to obtain the conditional characteristic function in the model. We have imposed an affine structure on the coefficients of the process, which leads to the closed form or nearly closed form expressions for the conditional characteristics functions.

3.2 Model Development.

Since standard stochastic models of modern finance found their way to the energy market, the prominent of all models geometric Brownian motion (GBM) can not be applied directly to electricity prices as it does not allow for price spikes, jumps and mean-reversion. To capture the mean-reversion and price spikes present in electricity prices we develop a mean reverting jump-diffusion model adjusted to incorporate almost all of the features of electricity price series under the study.

We start with specifying the logarithm of the spot price process, lnS_t, can be written as

$$\ln S_t = X_t, \tag{3.1}$$

such that the spot price S_t is given by

$$S_t = e^{X_t}$$

where X_t is a stochastic process whose dynamics is given by

$$dX_t = \mu(X_t, t)dt + \sigma dW_t + Q_t dP_t(\omega) \tag{3.2}$$

But the drift term in the mean-reverting model is governed by the distance between the current price X_t and the mean reversion level β as well as mean reversion rate κ, that is;

$$\mu(X_t, t) = \beta - \kappa X_t = \kappa(\frac{\beta}{\kappa} - X_t) \tag{3.3}$$

If the spot price is below the mean-reversion level, the drift will be positive, resulting to an upward influence on the spot price. This means that, when prices are relative low, supply will decrease since some of the higher cost producers will exit the market, putting upward pressure on prices. And if it is above the mean reversion level, the drift will be negative, exerting a downward influence on the spot price. That is, supply will increase since higher cost producers of the commodity will enter the market putting a downward pressure on prices. Over time, this results in a price path that is determined by the mean-reversion level at a speed determined by the mean reversion rate κ.
Substituting $\frac{\beta}{\kappa} = L$
we obtain the following SDE;

$$dX_t = \kappa(L - X_t)dt + \sigma dW_t + Q_t dP_t(\omega). \tag{3.4}$$

Where, L is the long-term mean which depends on seasonality function, k is the mean reversion rate, σ is the volatility term responsible for randomness of the process that is set to a constant, W_t is a standard Brownian motion with $dW_t \sim N(0, dt)$ for an infinitesimal time interval dt, which is responsible for small fluctuations around the long term mean for mean reverting processes, and P_t is a discontinuous, one dimensional standard Poisson process with arrival rate ω. During dt,

$$dP_t = \begin{cases} 1 & \text{if there is jump} \\ \\ 0 & \text{otherwise} \end{cases}$$

The jump amplitude Q_t is exponentially distributed with mean γ and the sign of the jump Q_t is distributed as a Bernoulli random variable with parameter ψ . We assume that the Brownian motion, Poisson process, and random jump amplitude are all Markov and pairwise independent.

3.3 Mathematical Description of Affine Jump Diffusion process.

Consider a filtered probability measure space $(\Omega, \mathcal{F}, \mathbb{F} := (\mathcal{F}_t)_{0 \leq t \leq T}, P)$, where t is "time" across a fixed trading interval $[0, T]$. Suppose that X is a Markov process relative to $\mathcal{F}_t$, taking values in a state space $D \subset \mathbb{R}^n$ and solving the stochastic differential equation

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + dZ_t. \tag{3.5}$$

In the integral form the SDE becomes ,

$$X_t = X_0 + \int_0^t \mu(X_s)ds + \int_0^t \sigma(X_s)dW_s + \sum_{i=1}^m Z_t$$

Described under the measure P, and driven by the $\mathcal{F}_t$ -adapted standard Brownian motion W in $\mathbb{R}^n$, with parameter functions $\mu : D \to \mathbb{R}^n$ and $\sigma : D \to \mathbb{R}^{n\times n}$. The process Z is a pure jump, with fixed jump amplitude distribution v on $\mathbb{R}^n$ and arrival intensity $\lambda(X_t)$, where $\lambda : D \to \mathbb{R}_+$. Assuming that μ, σ and λ are regular enough such that the above equation has strong solution (in sense of Karatzas and Shreve (1999), section 5.2). The process for X is "affine" when the drift vector μ, "instantaneous" covariance matrix $\sigma\sigma^T$ and jump intensity λ have affine dependence on X:

$$\mu(X_t) = K_0 + K_1 X_t \quad : \quad (K_0, K_1) \in \mathbb{R}^n \times \mathbb{R}^{n\times n}$$

$$\sigma(X_t)\sigma(X_t)^T = H_0 + H_1 X_t \quad : \quad (H_0, H_1) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n\times n} \tag{3.6}$$

$$\lambda(X_t) = l_0 + l_1 X_t \quad : \quad (l_0, l_1) \in \mathbb{R}^{n(n+1)} \times \mathbb{R}^{n(n+1)\times n}$$

Given an initial condition X_0, the tuple $\theta = (K_0, K_1, H_0, H_1, l_0, l_1)$ can be used to determine a transform $\Psi_\theta : \mathbb{C}^n \times [0, \infty) \times [0, \infty) \times D \to \mathbb{C}$ of X_T conditional on $X_t, 0 \le t \le T$ defined by

$$\Psi_\theta(u, t, T, X_t) = E^\theta[\exp(\mu.X_T)|X_t], \tag{3.7}$$

Where E^θ denotes the expectation under the distribution of X_T determined by θ .Duffie *et al* [18] have proved that if we suppose that $\theta = (K_0, K_1, H_0, H_1, l_0, l_1)$ is well behaved at $(\mathbf{u}, \mathbf{T})$, the transform Ψ_θ of $X_t, 0 \le t \le T$,defined by (3.7) , exists and is given by

$$\Psi_\theta(\mathbf{u}, \mathbf{t}, \mathbf{T}, \mathbf{X_t}) = \exp(\mathbf{A}(\mathbf{u}, \mathbf{t}, \mathbf{T}) + \mathbf{B}(\mathbf{u}, \mathbf{t}, \mathbf{T}).\mathbf{X_t}). \tag{3.8}$$

$A(.)$ and $B(.)$ satisfy the following complex-valued Riccati equations,

$$\begin{aligned} \frac{\partial A(u,t,T)}{\partial t} &= -K_0.B(t) - \frac{1}{2}B(t)^T H_0 B(t) - l_0(\varphi(B(t)) - 1), \quad A(u,T,T) = 0, \\ \frac{\partial B(u,t,T)}{\partial t} &= -K_1^T.B(t) - \frac{1}{2}B(t)^T H_1 B(t) - l_1(\varphi(B(t)) - 1), \quad B(u,T,T) = u \end{aligned} \tag{3.9}$$

For $c \in \mathbb{C}$, the complex n-tuples define "the jump transform" $\phi(c) = \int_{R^n} \exp(c.z)dv(z)$, which is assumed to be known in closed-form, whenever the integral is well defined. For example, for normally distributed jump size with mean μ and variance σ^2 , it can be shown that $\varphi(c) = \exp(\mu c + \frac{1}{2}\sigma^2 c^2)$; for an exponentially distributed jump size with mean μ ,

$$\varphi(c) = \frac{1}{1 - \mu c}. \tag{3.10}$$

By setting $\mathbf{u} = \mathbf{is}(\mathbf{i} = \sqrt{-\mathbf{1}})$, we can obtain the CCF of X_T conditional to X_t as:

$$\begin{aligned} \phi(s, \theta, X_T | X_t) &:= \Psi_\theta(is, t, T, X_t) \\ &= E^\theta[\exp(is.X_T)|X_t] \\ &= \int_{R^n} \exp(is.X_T) f(X_T, \theta | X_t) dX_t, \end{aligned} \tag{3.11}$$

where $f(X_T, \theta|X_t)$ is the conditional density function of X_T conditional on X_t.
Notice that the Conditional Characteristic Function (CCF) is actually the Fourier transform of the conditional density function. Through the inverse Fourier transform, we can recover the conditional density function from the CCF and implement a usual Maximum Likelihood (ML) estimation.

Observe that the equation (3.4) fits in the framework presented in equation (3.6) with $K_0 = \kappa L, \quad K_1 = -\kappa, \quad H_0 = \sigma^2, \quad H_1 = 0, \quad l_0 = \omega, \quad l_1 = 0.$
Thus, the CCF of X_T given $X_t, \phi(s, \theta, X_T|X_t)$ for the equation (3.4), takes the form of

$$\begin{aligned} \phi(s, \theta, X_T | X_t) &= E[\exp(isX_T)|X_t], \\ &= \exp(M(s, t, T, \theta) + N(s, t, T, \theta)X_t), \end{aligned} \tag{3.12}$$

where $M(.)$ and $N(.)$ satisfy the following system of complex-valued ordinary differential equations (ODE):

$$\frac{\partial M(s,t,T,\theta)}{\partial t} = -\kappa L N(s,t,T,\theta) - \frac{1}{2}\sigma^2 N^2(s,t,T,\theta) - \omega(\varphi(N(s,t,T,\theta)) - 1), \tag{3.13}$$

$$\frac{\partial N(s,t,T,\theta)}{\partial t} = \kappa N(s,t,T,\theta),$$

with boundary conditions:

$$M(s,T,T,\theta) = 0, \quad N(s,T,T,\theta) = is. \tag{3.14}$$

Here the "jump transform" $\varphi(N(s,t,T,\theta))$ is given by

$$\varphi(s,t,T,\theta) = \psi \int_0^\infty \exp(N(s,t,T,\theta)z)\frac{1}{\gamma}\exp(-\frac{z}{\gamma})dz$$

$$+(1-\psi)\int_0^\infty \exp(-N(s,t,T,\theta)z)\tfrac{1}{\gamma}\exp(-\tfrac{z}{\gamma})dz, \tag{3.15}$$

$$= \frac{\psi}{1 - N(s,t,T,\theta)\gamma} + \frac{1-\psi}{1 + N(s,t,T,\theta)\gamma}.$$

Then, we solve the system (3.13) for $M(.)$ and $N(.)$ and applying the corresponding boundary conditions we obtain

$$M(s,t,T,\theta) = iLs(1 - e^{-\kappa(T-t)}) - \frac{\sigma^2 s^2}{4\kappa}(1 - e^{-2\kappa(T-t)})$$

$$+\tfrac{i\omega(1-2\psi)}{\kappa}(\arctan(\gamma s e^{-\kappa(T-t)}) - \arctan(\gamma s)) + \tfrac{\omega}{2\kappa}\ln(\tfrac{1+\gamma^2 s^2 e^{-2\kappa(T-t)}}{1+\gamma^2 s^2}) \tag{3.16}$$

$$N(s,t,T,\theta) = ise^{-\kappa(T-t)}.$$

3.4 Parameter Estimation.

Affine processes are flexible enough to allow us to capture the special characteristics of electricity prices such as mean-reversion, seasonality, and spikes [16]. Under suitable regularity conditions, we can explore the information from the CCF of discretely sampled observations to develop computationally tractable and asymptotically efficient estimators of the parameters of affine processes [18]. Also, the CCF is unique and contains the same information as the conditional density function through the Fourier transform. Through the approach of ML-CCF estimation, we can use CCF to recover the conditional density function via the Fourier transform and implement a usual ML estimation. If the N-dimensional state variables are all observable, ML-CCF estimation can be implemented to obtain ML-CCF estimators that are asymptotically efficient [43].

However, the estimation can be costly in higher dimensions ($N \geq 2$) because we need to compute the multivariate Fourier inversions repeatedly and accurately in order to maximize the likelihood function. According to Singleton (2001), considerable computational saving can be achieved by using limited-information ML-CCF (LML-CCF) estimation [43]. Suppose $\{X_t, t = 1, 2, ...\}$ is a set of discretely sampled observations of an N-dimensional state variable with a joint CCF $\phi(s, \theta, X_{t+1}|X_t)$ Let η_j denote a N-dimensional selection vector where the j^{th} entry is 1 and zeros elsewhere. Define $X^j_{t+1} := \eta_j.X_{t+1}$, then the conditional density of X^j_{t+1} conditioned on X_t is the inverse Fourier transform of $\phi(\xi\eta_j, \theta, X_{t+1}|X_t)$ with some scalar ξ^2

$$f_j(X^j_{t+1}, \theta|X_t) = \frac{1}{2\pi}\int_{\mathbb{R}} \phi(\xi\eta_j, \theta, X_{t+1}|X_t)e^{-i\xi\eta_j' X_{t+1}}d\xi. \tag{3.17}$$

The basic idea behind this is to exploit the information in $f_j(X^j_{t+1}, \theta|X_t)$ instead of the information in the joint conditional density function,

$$f(X_{t+1}, \theta|X_t) = \frac{1}{(2\pi)^N}\int_{\mathbb{R}^N} \phi(s, \theta, X_{t+1}|X_t)e^{-is' X_{t+1}}ds. \tag{3.18}$$

Thus, the estimation involves at most N one-dimensional integrations instead of doing a N-dimensional integration. The estimators obtained are called LML-CCF estimators. Although the LML-CCF estimators do not exploit any information about the joint conditional density function, they are typically more efficient than the quasi maximum likelihood (QML) estimators for affine diffusions [43].

3.4.1 ML-CCF Estimators.

ML estimation is the most common method of estimating the parameters of stochastic processes if the probability density has an analytical form. It provides a consistent approach to parameter estimation problems and ML estimators become minimum variance unbiased estimators as the sample size increases. Suppose that X is a N-dimensional continuous random variable with probability density function $f(X, \theta)$ where $\theta = \{\theta_1, ...\theta_k\}$, are k unknown constant parameters which need to be estimated.

Thus, given a sequence of observations $\{X_t\}$ sampled at $t = 1, 2, ..., n$, the log likelihood function at the sample is given by:

$$\mathcal{L}(X_1, ..., X_n, \theta) = \sum_{t=1}^{n} \ln f(X_t, \theta). \tag{3.19}$$

The maximum likelihood based estimators of θ are obtained by maximizing $\mathcal{L}(.)$ is

$$\hat{\theta}_{ml} = \arg_\theta max \mathcal{L}(X_1, ..., X_n, \theta) = \arg_\theta max \sum_{t=1}^{n} \ln f(X_t, \theta). \tag{3.20}$$

Now, for the model equations with N-dimensions, the CCF,$\phi(s, \theta, X_{t+1}|X_t)$, of the sample is known, often in closed-form, as an exponential of an affine function of X_t . Thus, the conditional density function of X_{t+1} given X_t can be obtained by Fourier transform of CCF:

$$f(X_{t+1}, \theta|X_t) = \frac{1}{(2\pi)^N} \int_{\mathbb{R}^N} \phi(s, \theta, X_{t+1}|X_t) e^{-is.X_{t+1}} ds. \tag{3.21}$$

We can use the standard ML estimation based on this conditional density function to obtain ML-CCF estimators of the sample as:

$$\hat{\theta}_{CCF} = \arg_{\theta} max \sum_{t=1}^{n} \ln(f(X_{t+1}, \theta | X_t)). \tag{3.22}$$

Therefore for the model equation (3.4), the conditional density function of X_{t+1} given X_t of the sample is given by,

$$f(X_{t+1}, \theta | X_t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(s, \theta, X_{t+1} | X_t) e^{-isX_{t+1}} ds, \tag{3.23}$$

$$= \tfrac{1}{2\pi} \textstyle\int_{-\infty}^{\infty} h(\theta, s) e^{isY_t} ds,$$

where

$$Y_t = (X_{t+1} - L) - e^{\kappa}(X_t - L), \tag{3.24}$$

and

$$h(\theta, s) = \exp(-\frac{\sigma^2 s^2}{4\kappa}(1 - e^{-2\kappa}) + \frac{i\omega(1-2\psi)}{\kappa}(\arctan(\gamma s e^{-\kappa}) - \arctan(\gamma s)) \tag{3.25}$$

$$+ \frac{\omega}{2\kappa} ln(\frac{1 + \gamma^2 s^2 e^{-2\kappa}}{1 + \gamma^2 s^2})).$$

To simplify the computation of the integral (3.23) we define

$$F(Y_t, \theta) := f(X_{t+1}, \theta | X_t) = \frac{1}{2\pi} \lim_{R \to \infty} \int_{-R}^{R} e^{isY_t} h(\theta, s) ds. \tag{3.26}$$

From the theory of continuity, we find that $|h(\theta, s)|$ is continuous in s and $|h(\theta, s)| \leq -\frac{\sigma^2 s^2}{4\kappa}(1 - e^{-2\kappa})$. Thus we can truncate the integral to a finite interval $[-R, R]$ outside of which the function $h(\theta, s)$ to be integrated is negligibly small. Therefore, for this choice of R,

$$F(Y_t, \theta) \approx \frac{1}{2\pi} \int_{-R}^{R} e^{isY_t} h(\theta, s) ds. \tag{3.27}$$

Also, if we discretize Y_t into N subintervals such that:

$$Y_m = m\Delta Y_t = m(\tfrac{Y_t}{N}),$$

$$s_k = k\Delta s = k(\tfrac{R}{N}),$$

then we have

$$F(Y_t, \theta) \approx \frac{1}{2\pi}\frac{R}{N}\sum_{m=-N}^{N-1}(e^{-imk\frac{RY_t}{N^2}}h(\theta, \frac{mR}{N})). \tag{3.28}$$

Also, if we make rearrangement that $\frac{RY_t}{N^2} = 2\pi$, we have

$$F(Y_t, \theta) \approx \frac{1}{2\pi}\frac{R}{N}\sum_{m=-N}^{N-1}(e^{-imk\frac{2\pi}{N}}h(\theta, \frac{mR}{N})). \tag{3.29}$$

Therefore, we can approximate $F(Y_t, \theta)$ by the discrete Fourier transform (DFT) of $h(\theta, \frac{mR}{N})$, and the integral in equation (3.23) can be estimated on a suitable grid of s values by a fast Fourier transform (FFT) algorithm.

CHAPTER 4

DATA ANALYSIS AND METHODOLOGY.

4.1 Source of Data.

The data used in this work were secondary data from Nord Pool energy market exchange. The data were daily average electricity spot prices from 4^{th} January, 1997 to 27^{th} April, 2000. Nord Pool is the power exchange in the Scandinavian countries (Sweden, Finland, Norway and Denmark) launched on private initiative, by a combination of generators, distributors and traders. Its genuine role is to match the supply and demand of electricity to determine a publicly announced market clearing price (MCP).

4.2 Data Description.

A summary of statistics for the daily average electricity price series from 4^{th} January, 1997 to 27^{th} April, 2000 is presented in Table 4.1. The statistics reported are for electricity prices (S_t), the change in electricity prices (dS_t), the logarithm of electricity prices ($\ln S_t$) and the log returns of electricity prices ($d\ln(S_t)$) from one day to the next. Clearly, the price series are quite volatile due to its non-storability nature since in the Scandinavian market, electricity is mainly generated by hydro resources, therefore is more affected by transmission constraints, seasonality and weather. Also, since shocks in demand and supply cannot be smoothed, electricity spot prices are extremely volatile and occasionally reach extremely high levels, commonly known as "spikes". The price series has a standard deviation of 24.51, have positive skewness and excess kurtosis of 1.23 and 4.46 respectively.

	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
S_t	134.72333	24.51294	1.23465	7.45696	69.28225	374.33271
dS_t	-0.05969	12.70256	3.24494	180.62769	-196.96721	251.79548
$\ln(S_t)$	4.88759	0.17573	0.17885	0.91188	4.23819	5.92515
$d(\ln(S_t))$	-0.00034	0.07158	2.13974	4.68811	-0.74693	1.11673

Table 4.1: Descriptive statistics for the daily average electricity spot prices.

Note: Here and the remainder of this dissertation, the column labeled Std.Dev reports the standard deviation. Skewness and Kurtosis are the third and fourth moments around the mean, namely $Skewness = \frac{E[X-E[X]]^3}{[var[X]]^{1.5}}$, $Kurtosis = \frac{E[X-E[X]]^4}{[var[X]]^2}$. For a normal distribution, Skewness is equal to 0 and Kurtosis is equal to 3. Thus, $ExcessKurtosis = Kurtosis - 3$.

Like demand spot electricity prices are not uniform throughout the week. The intra-day and intra-week variations caused by different levels of working activities translate into periodical fluctuations in electricity prices. Our task is not to address the issue of intra-day and intra-week variations, but we analyze only the daily average prices. The daily price time series of the chosen data is displayed in Figure 4.1 and 4.2 for log-prices, where from a first visual inspection one can realize all the stylized facts described before (volatility, mean reversion, seasonality to some extent and spikes).

As it has been well documented due to its importance in electricity prices, mean reversion is also quite clear in the Figure 4.1 where price oscillates around the mean level and whenever there is a jump the price is pulled back to this mean level rapidly after a jump or spike. But the mean reversion is not very large because of hydro generation of

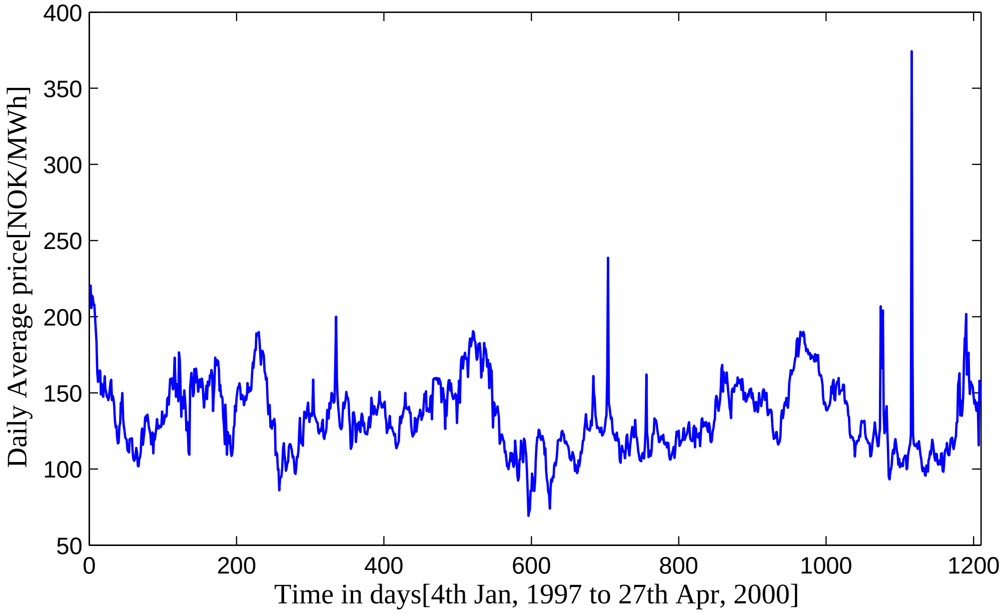

Figure 4.1: Daily average electricity spot price since 4th January, 1997 until 27th April, 2000 (1210 observations).

power, where hydro reservoirs play the important of indirect storage of electricity. Also volatility in prices is very high which can be shown very clearly in logarithm of prices (log (prices)) as indicated in Figure 4.2 below.

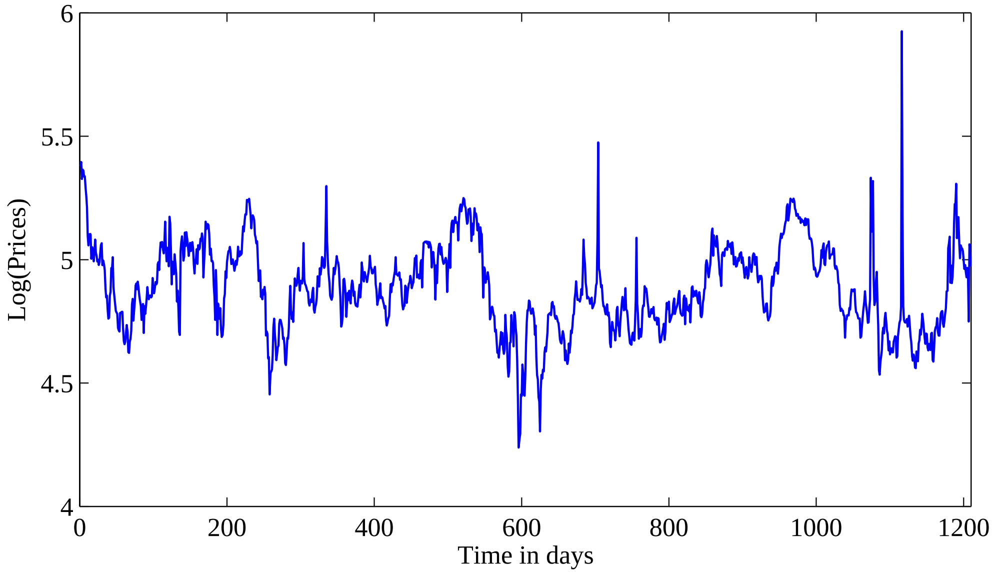

Figure 4.2: Logarithm of electricity prices which reveals properly the main features of electricity market.

We can also see that the descriptive statistics for the returns shows that the prices returns do not follow normal distribution, which is a crucial assumption for most financial theories and models. Also the evidence that, there is high departure from normality of the data is provided by normality test in Figure 4.8.

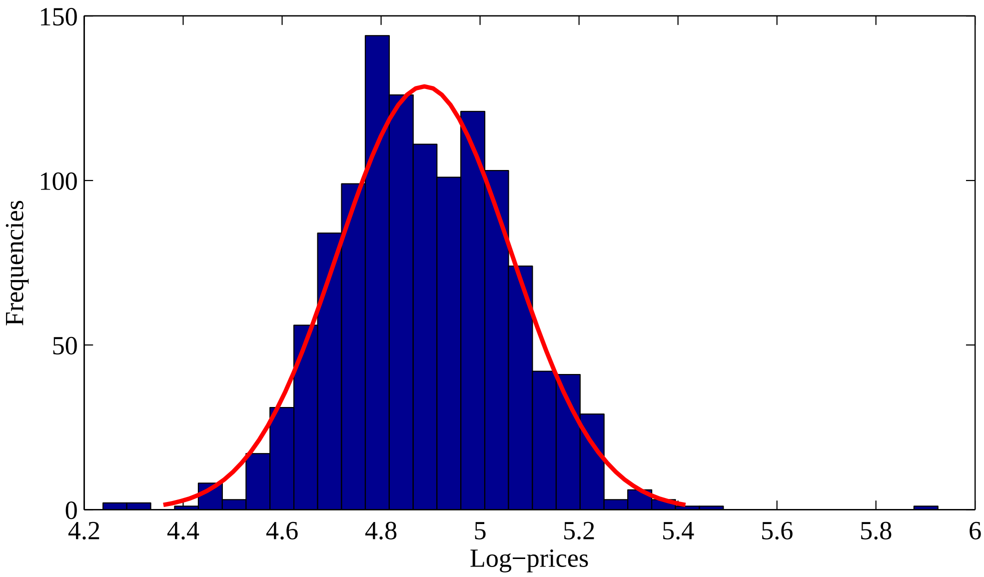

Figure 4.3: Histogram for logarithm of spot prices showing distribution of log-prices for the given period of time, superimposed is the theoretical normal curve.

From the histogram in Figure 4.3 one can visualize that the log prices are slightly skewed to the right, implying that for some days the prices at Nord Pool were extremely high for the underlying period, which shows the existence of jumps as shown in Figure 4.1 above.

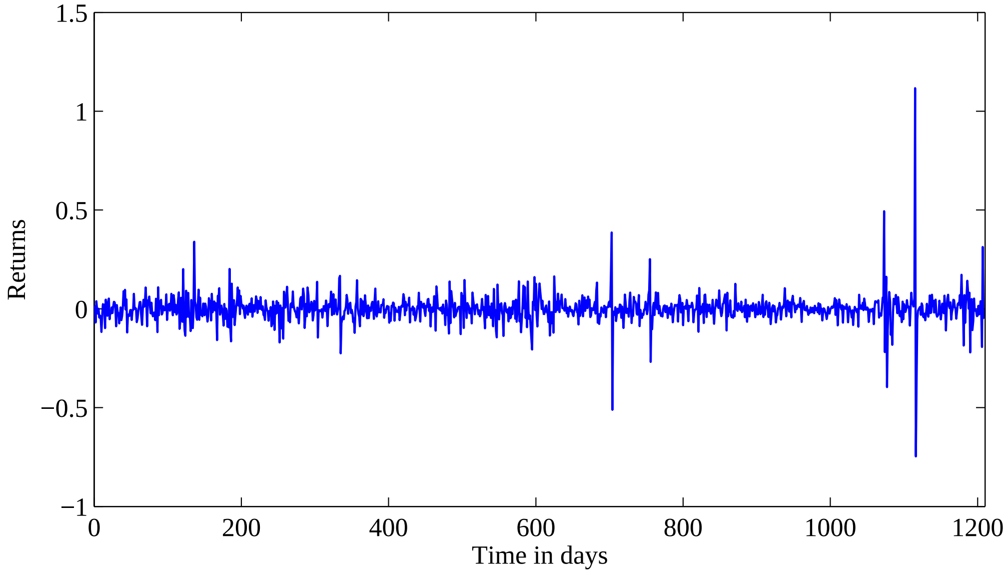

Figure 4.4: Log-returns price series showing few price spikes.

Since we are working on the daily scale spikes typically do not last more than one time point (i.e. one day), according to Weron (2006), positive jump is followed by a negative of approximately the same magnitude as shown in Figure 4.4.

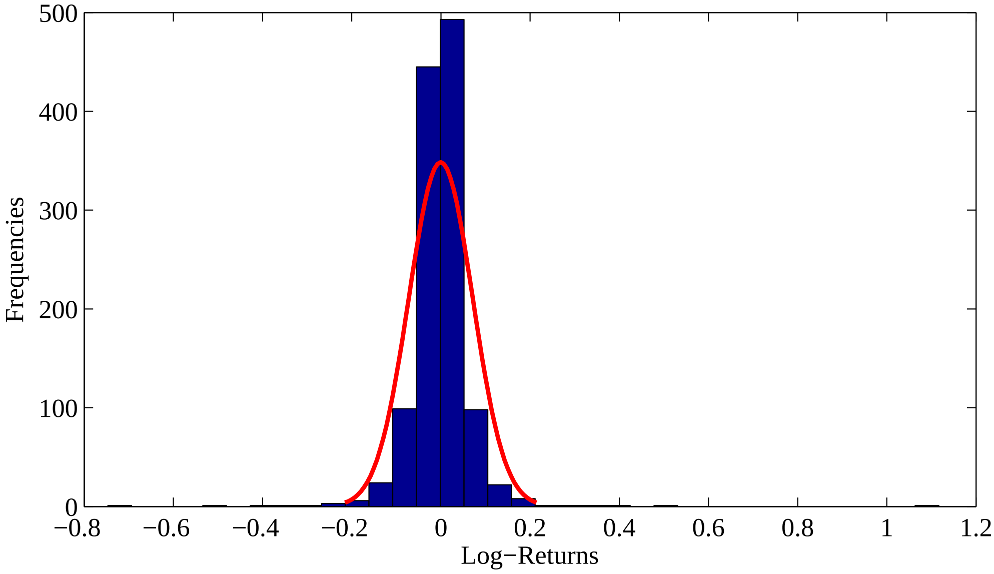

Figure 4.5: Histogram showing distribution of price returns superimposed with a theoretical normal curve .

4.3 Normality Tests.

In the Black-Scholes model the stock prices are assumed to be log-normally distributed, which is equivalent to saying that the returns of the prices have a Gaussian or Normal distribution.

For electricity though, the departure from Normality is more extreme. Figure 4.8 shows a Normality test for returns of the electricity spot price from 4/01/97 to 27/04/00. A solid line connects the 25^{th} and 75^{th} percentiles in the data and a dashed line extends it to the ends of the data. If the returns were indeed normally distributed the graph would be a straight line. That is all the data points would fall near the line. We can clearly observe that this is not the case, as evidenced from the fat tails. For instance, corresponding to a probability of 0.006 we have returns which are higher than 0.2; instead if the data were perfectly normally distributed, the dotted lines suggests the probability of such returns should be virtually zero.

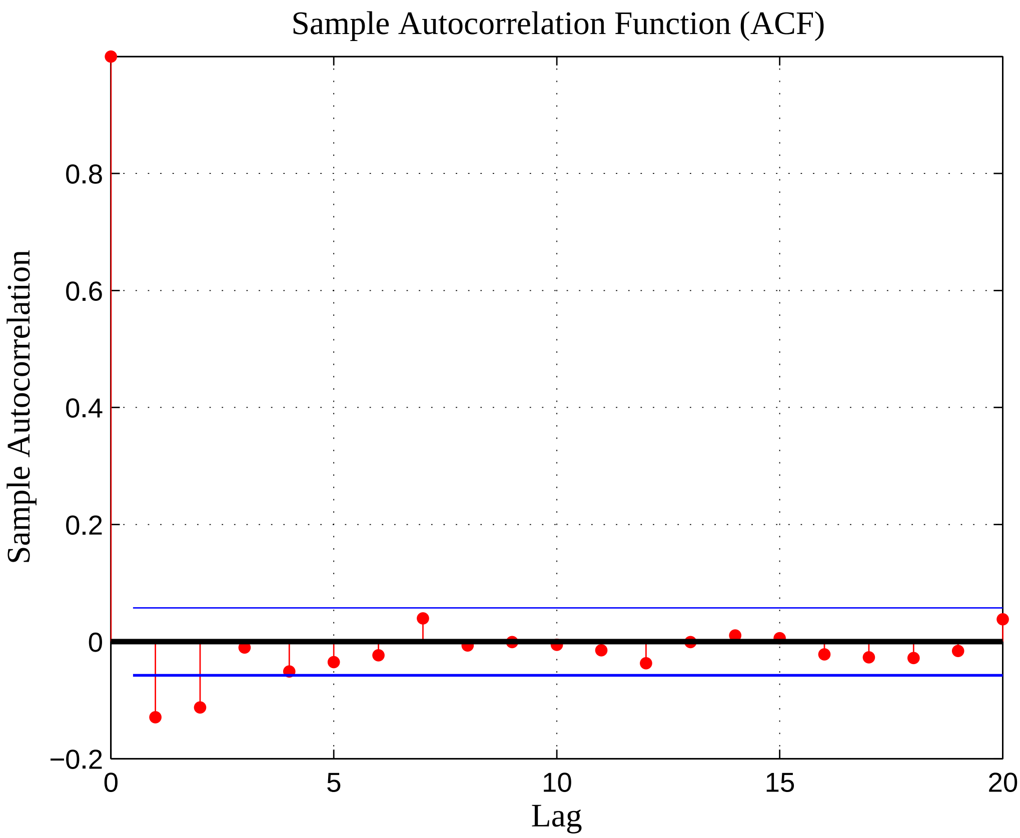

Figure 4.6: ACF for price return series showing some important lags.

where most of the values fall within the bounds. There is no clear seasonality that can be observed from the lags.

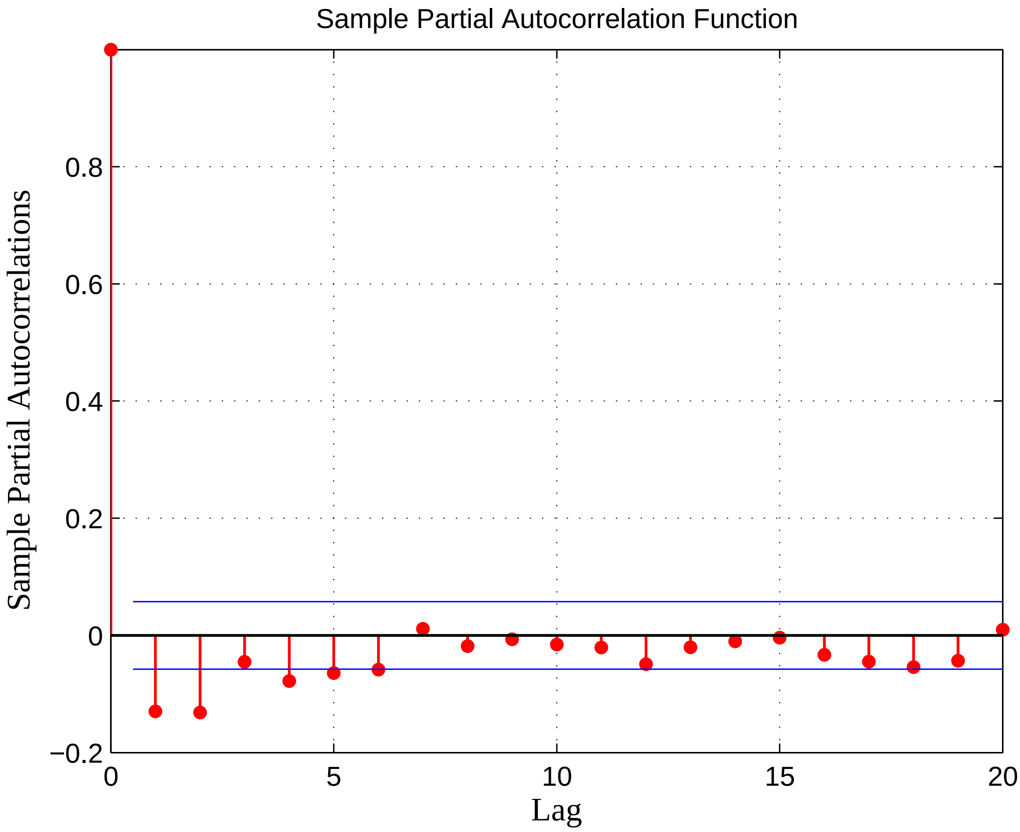

Figure 4.7: PACF for price return series, where most of the values are within the bounds.

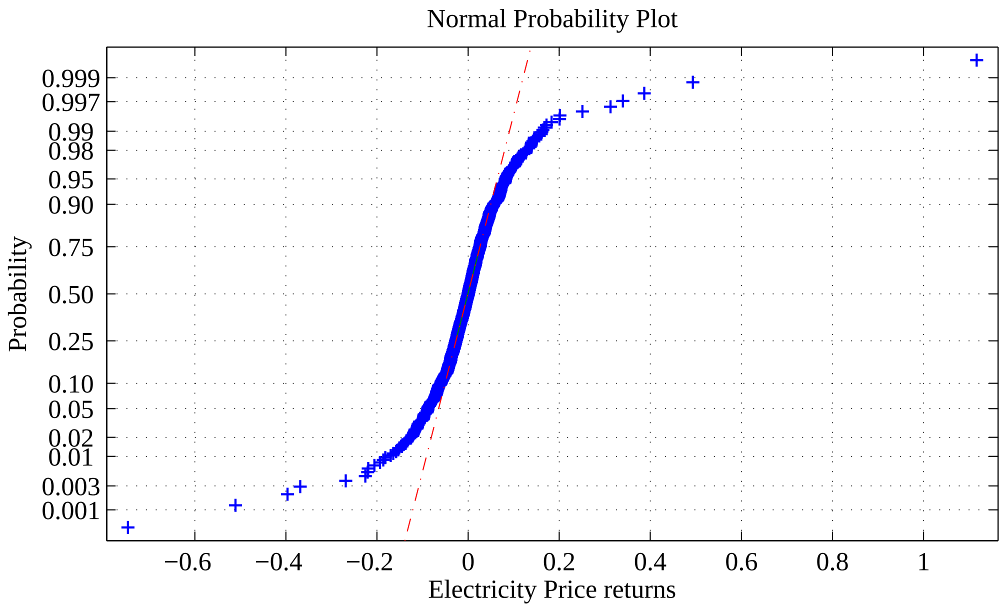

Figure 4.8: Normal probability test for returns of electricity prices from 4/01/97 to 27/04/00.

4.4 Calibration of the model.

The calibration of Mean-reverting jump diffusion models is related to a more general way of estimating the parameter of continuous-time jump diffusion processes from discretely sampled data. In this section we present estimates from fitting the model (3.4) using the methodology described in section 3.4. In calibrating the model the daily electricity prices were transformed to returns (log-returns). For this purpose Matlab function *price2ret.m* was used. It constructs so-called log-returns, which means

$$r_t = \ln \frac{S_t}{S_{t-1}}, \tag{4.1}$$

where

- r_t is the return for the time t,
- S_t is the electricity price at time t,

Parameter	Estimate	Standard Error	T Statistic
κ	0.11396	0.0040	28.1000
L	-0.00792	0.0014	-5.8573
σ	0.16453	0.0674	0.4015
ω	0.85146	0.0502	16.9664
ψ	0.12146	0.0223	5.4381
γ	0.33674	0.0058	58.0517

Table 4.2: Daily Electricity Price Returns parameters values for the model.

- S_{t-1} is the electricity price at time $t-1$.

For the model developed, we perform calibration via maximum likelihood (ML) estimation based on Fourier inversion of the conditional characteristics function (CCF). As we have computed the CCF of given for the model in section 3.4, the estimates are obtained by using standard ML estimation on this conditional density. We used Matlab software to estimate the parameter used in the model by performing optimization of the log-likelihood function of the conditional density. The resulting parameter estimates, their corresponding standard errors and T-statistic for the daily electricity price returns are provided in the Table 4.2.

All estimates are daily based. The standard errors are quite small, and the absolute values of the T-Statistic are all larger than 1.96 except for sigma value, which indicates that the parameters are likely significant using the 95% confidence interval. The mean reverting rate $kappa(\kappa)$ is approximately 0.114, thus the half-life of the mean-reverting process which is the key property of the mean-reverting process is approximately 6.1days.

This means that it takes 6.1days for the price to revert half way back to its long-term level from its current level assuming no more random shocks arrive.

4.5 Descriptive Statistics of Empirical Daily Returns Vs Calibrated Daily Returns.

Given the CCF, we can obtain the moments for any choice of jump distribution where the jump intensity or distribution does not depend on the stable variables. To obtain the moments, we differentiate the CCF (ϕ) successively with respect to s and then find the value of the derivative when $s = 0$. Let U_n denote the n^{th} moment, and ϕ_n be the n^{th} derivative of ϕ with respect to s, that is

$$\phi_n = \frac{\partial^n \phi}{\partial s^n} \tag{4.2}$$

Then

$$U_n := \frac{1}{i^n}[\phi_n | s = 0]. \tag{4.3}$$

Therefore, for the model we have developed in equation 3.4 the CCF of X_T conditional to X_t has the closed form

$$\begin{aligned} \phi(s, \theta, X_T | X_t) &= E[\exp(isX_T | X_t] \\ &= \exp(M(s, t, T, \theta) + N(s, t, T, \theta) X_t). \end{aligned} \tag{4.4}$$

where

$$M(s,t,T,\theta) = iLs(1-e^{-\kappa(T-t)}) - \frac{\sigma^2 s^2}{4\kappa}(1-e^{-2\kappa(T-t)})$$

$$+\tfrac{i\omega(1-2\psi)}{\kappa}(\arctan(\gamma s e^{-\kappa(T-t)}) - \arctan(\gamma s)) + \tfrac{\omega}{2\kappa}\ln(\tfrac{1+\gamma^2 s^2 e^{-2\kappa(T-t)}}{1+\gamma^2 s^2}) \tag{4.5}$$

$$N(s,t,T,\theta) = ise^{-\kappa(T-t)}.$$

Then the unconditional (stationary) characteristics function $\phi(s,\theta)$ of X_T , which is obtained by letting $T \to \infty$, is of the form

$$\phi(s,\theta) = \exp(iLs - \frac{\sigma^2 s^2}{4\kappa} + \frac{i\omega(2\psi-1)}{\kappa}\arctan(\gamma s) - \frac{\omega}{\kappa}\ln(1+\gamma^2 s^2)). \tag{4.6}$$

We let X_∞ denote the random variable with this unconditional (stationary) distribution. Then we obtain the n^{th} moment by the equation (4.2) as follows:
To get the first moment, we differentiate $\phi(s,\theta)$ with respect to s,

$$\phi_1 := \frac{\partial\phi(s,\theta)}{\partial s} = (iL - \frac{\sigma^2 s}{2\kappa} + \frac{i\omega(2\psi-1)}{\kappa}\frac{\gamma}{1+\gamma^2 s^2} - \frac{\omega}{2\kappa}\frac{2\gamma^2 s}{1+\gamma^2 s^2})\phi(s,\theta). \tag{4.7}$$

Thus, the first moment U_1 is given by

$$\begin{aligned} U_1 := E[X_\infty] &= \frac{1}{i}[\phi_1 | s=0] \\ &= L + \frac{\omega(2\psi-1)}{\kappa}\gamma. \end{aligned} \tag{4.8}$$

Similarly, we can obtain other moments as follows:

$$U_2 := E[X_\infty^2] = \frac{\sigma^2 + 2\omega\gamma^2}{2\kappa} + U_1^2,$$

$$U_3 := E[X_\infty^3] = \frac{2\omega(2\psi - 1)\gamma^3}{\kappa} + 3U_1U^2 - 2U_1^3, \tag{4.9}$$

$$U_4 := E[X_\infty^4] = \frac{6\omega\gamma^4}{\kappa} + 3U_2^2 - 12U_2U_1^2 + 4U_3 + 6U_1^4.$$

Therefore, we have the unconditional variance, skewness and kurtosis as follows:

$$Variance := E[(X_\infty - U_1)^2] = \frac{\sigma^2 + 2\omega\gamma^2}{2\kappa},$$

$$Skewness := E\left[\frac{(X_\infty - U_1)^3}{(U_2 - U_1^2)^{1.5}}\right] = \frac{4\sqrt{2\kappa}\omega\gamma^3(2\psi - 1)}{(\sigma^2 + 2\omega\gamma^2)^{1.5}}, \tag{4.10}$$

$$Kurtosis := E\left[\frac{(X_\infty - U_1)^4}{(U_2 - U_1^2)^2}\right] = \frac{24\kappa(\omega\gamma^4)}{(\sigma^2 + 2\omega\gamma^2)^2} + 3.$$

We can also obtain the conditional variance, skewness and Kurtosis for the process in the same way as follows:

$$Variance := E[(X_{t+1} - U_1)^2|X_t] = (\frac{\sigma^2 + 2\omega\gamma^2}{2\kappa})(1 - e^{-2\kappa}),$$

$$Skewness := E\left[\frac{(X_{t+1} - U_1)^3}{(U_2 - U_1^2)^{1.5}}|X_t\right] = \frac{4\omega\sqrt{2\kappa}\gamma^3(2\psi - 1)}{((\sigma^2 + 2\omega\gamma^2)(1 - e^{-2\kappa}))^{1.5}}(1 - e^{-3\kappa}), \tag{4.11}$$

$$Kurtosis := E\left[\frac{(X_{t+1} - U_1)^4}{(U_2 - U_1^2)^2}|X_t\right] = \frac{24\kappa\omega\gamma^4(1 + e^{-2\kappa})}{(\sigma^2 + 2\omega\gamma^2)^2(1 - e^{-2\kappa})} + 3.$$

From this analysis, we find that higher moments (Variance, Skewness and Kurtosis) do not include the information of the long term mean, L ; they contain the information on

	Empirical results		Theoretical results	
	Unconditional	Conditional	Unconditional	Conditional
Std.Dev	0.17573	0.07158	0.1120	0.0510
Skewness	0.17885	2.13974	−0.4550	1.4319
Kurtosis	0.91188	4.68811	3.6777	4.1986

Table 4.3: Empirical results vs theoretical results for the model.

volatility of the prices and the jumps.

4.6 Simulation Results for the Model.

Using the parameters we obtained in Table 4.2, we can simulate the daily price series to compare with daily Electricity Prices. If we consider the jump conditions set, we can assume that the jumps are very few in our data set almost two distinct jumps in which $dP = 1$ for existence of jumps and $dP = 0$ for no jumps. To simulate the model we have taken the assumption that $dP = 0$ which is responsible for no jumps. Here we display the sample plots of simulated log prices in Figure 4.9, simulated price path (red line) and the sample plot of Daily Electricity Prices (blue line) in Figure 4.10. Notice that most of the empirical data except for those extreme "spikes" are inside the simulation range. Also the simulated prices have very high volatility due to the fact that the value of estimated sigma was not so significant compared to other parameters. From the simulated prices the fair price of energy for both buyers and producers is taken from the long-term mean L of the prices. For price returns $L = -0.00792$ as estimated from the model, then for the log-prices $L = 4.7936$. Thus the long-term mean for the spot prices is 120.73523. This is the price which is considered to be fair since the price dynamics mean revert around this value, the speed of the mean of the mean reversion is determined by the mean reversion rate $\kappa(kappa) = 0.11369$.

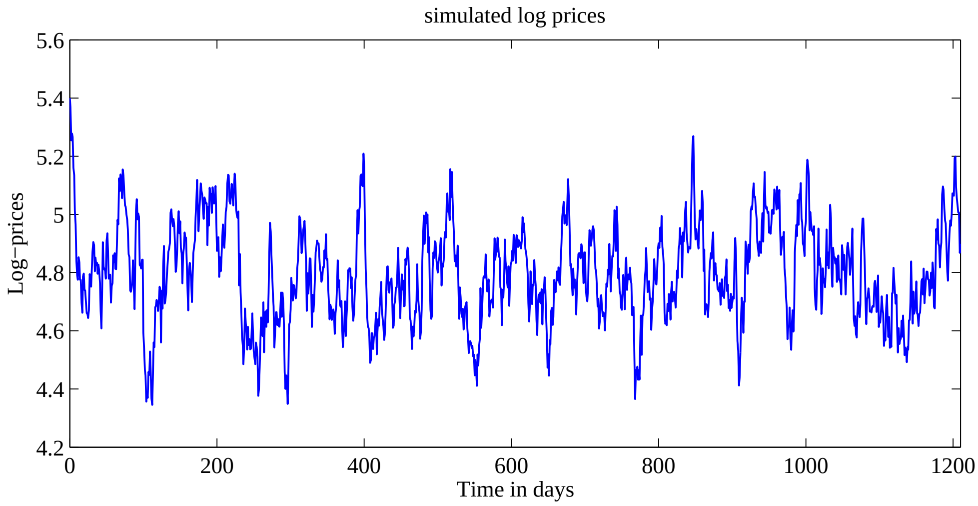

Figure 4.9: Simulation results for logarithm of Prices.

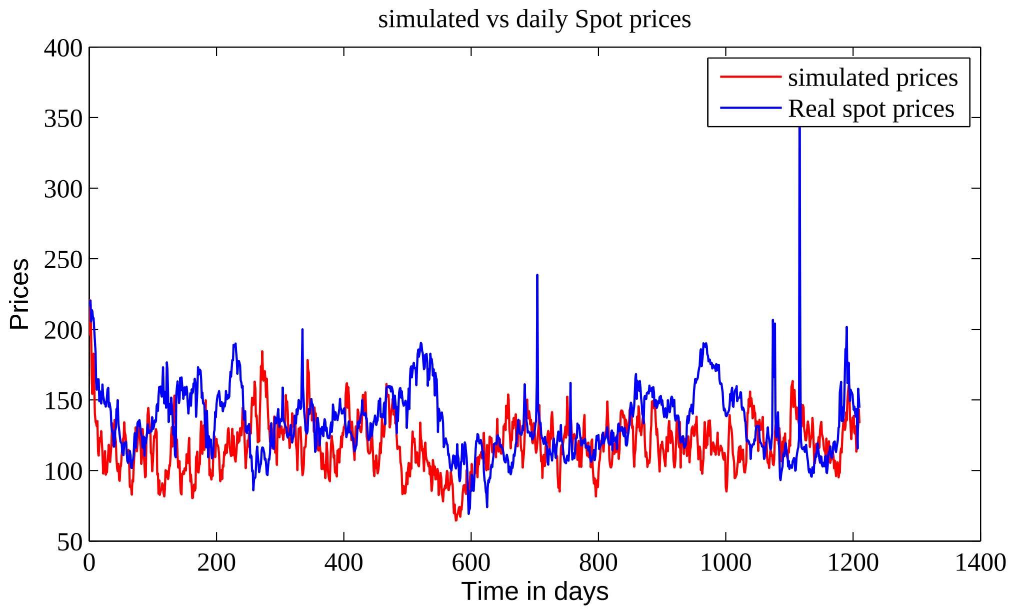

Figure 4.10: Simulated Energy Spot Prices versus Real Prices.

4.7 Forward Price.

As discussed earlier electricity is a very expensive commodity to keep in storage. Once purchased, it must be consumed almost immediately. Therefore, the usual hedging strategies adopted in case of other financial assets, like holding certain amounts of the underlying, which in this case is electricity, is not a pragmatic solution. Consequently, in the electricity market forwards on the spot prices are typically used as a hedging strategy.

The price at time t of the forward expiring at time T is obtained as the expected value of the spot price at expiry under an equivalent Q-martingale measure, conditional on the information set available up to time t; namely

$$F(t,T) = \mathbb{E}_t^Q[S_T|\mathcal{F}_t] \tag{4.12}$$

Thus, we need to replace $X_t = \ln S_t$ and integrate the resulting SDE in order to extract S_T and later calculate its expectation.

Regarding the expectation, we must calculate it under an equivalent Q-martingale measure. In a complete market this measure is unique, ensuring only one arbitrage-free price of the forward. However, in incomplete markets (such as the electricity market in our case) this measure is not unique, thus we are left with the difficult task of selecting an appropriate measure for the particular market in question. Another approach, common in the literature, is simply to assume that we are already under an equivalent measure, and thus proceed to perform the pricing directly. This latter approach would rely however in calibrating the model through implied parameters from a liquid market. This is certainly difficult to do in young markets and those which are about to be initiated, as there will be no liquidity of instruments that would enable us to do this. But for the case of Nordpool where our reference data have been taken, this is possible as the liquidity nature of the market exist.

We follow the approach by Schwartz' and Lucia in [42], which consists of incorporating a market price of risk in the drift, such that

$$\hat{L} \equiv L - \lambda^* \text{ and } \lambda^* \equiv \lambda \tfrac{\sigma}{\kappa}$$

where λ denotes the market price of risk per unit risk linked to the state variable X_t . This market price of risk, to be calibrated from market information, pins down the choice of one particular martingale measure. Under this measure now we may then rewrite the stochastic process in (3.4) for X_t as

$$dX_t = \kappa(\hat{L} - X_t)dt + \sigma d\hat{W}_t + \ln J dq_t \tag{4.13}$$

where the long term mean is assumed to have some seasonality factor $g(t)$, that is

$$\hat{L} = \tfrac{1}{\kappa}\tfrac{dg}{dt} + g(t) - \lambda\tfrac{\sigma}{\kappa},$$

and $d\hat{W}_t$ is the increment of Brownian motion (Wiener Process) in the Q-measure specified by the choice of λ . Also $\ln J = Q_t$ is the jump parameter associated with the Q-martingale measure. Integrating the process (4.13) after introducing the market price of risk we get

$$X_T = g(T) + (X_t - g(t))e^{-\kappa(T-t)} - \lambda \int_t^T \sigma e^{-\kappa(T-s)} ds$$
$$+ \int_t^T \sigma e^{-\kappa(T-s)} d\hat{W}_s + \int_t^T e^{-\kappa(T-s)} \ln J dq_s. \tag{4.14}$$

Since $S_T = e^{X_T}$,we can replace (4.14) and then substitute into (4.12) to get the forward price

$$F(t,T) = \mathbb{E}_t[S_T|\mathfrak{F}_t]$$

$$= e^{-\lambda \int_t^T \sigma e^{-\kappa(T-s)} ds} G(T)(\frac{S(t)}{G(t)})e^{-\kappa(T-t)} \mathbb{E}_t[e^{\int_t^T \sigma e^{-\kappa(T-s)} d\hat{W}_s}|\mathfrak{F}_t]\mathbb{E}_t[e^{\int_t^T e^{-\kappa(T-s)} \ln J dq_s}|\mathfrak{F}_t]. \tag{4.15}$$

In order to evaluate the first expectation above we make use of Ito's Isometry theory and probability theory, which is stated in **theorem 1** below.

Theorem 1 *If f belongs to $H_2[0,T]$, the space of random functions defined for all t in $[0,T]$, and $\int_0^T E(f(t))^2 dt < \infty$,then*

$E[\int_0^T f(t) dW_t] = 0$

and, $E[(\int_0^T f(t) dW_t)^2] = \int_0^T E[f(t)]^2 dt.$

Then the expectation is obtained as

$$E_t[e^{\int_t^T \sigma e^{-\kappa(T-s)} d\hat{W}_s}|\mathfrak{F}_t] = e^{\frac{\sigma^2}{2} \int_t^T e^{-\kappa(T-s)} ds} = e^{\frac{\sigma^2}{4\kappa}[1-e^{-2\kappa(T-t)}]}. \tag{4.16}$$

It follows that, the above equation (4.15) will be written as,

$$F(t,T) = e^{-\lambda \int_t^T \sigma e^{-\kappa(T-s)} ds} G(T)(\frac{S(t)}{G(t)})e^{-\kappa(T-t)} e^{\frac{\sigma^2}{4\kappa}[1-e^{-2\kappa(T-t)}]} \mathbb{E}_t[e^{\int_t^T e^{-\kappa(T-s)} \ln J dq_s}|\mathfrak{F}_t]. \tag{4.17}$$

T(Years)	5	3	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{12}$
F(0 ,T)	57.5753	70.2344	86.7926	96.9444	98.3469	98.5930	99.8283

Table 4.4: Current Forward prices with different expiries.

The remaining expectation in the forward expression (4.17) is taken under the risk neutral measure. Thus we have (detailed calculations will be shown in the appendix),

$$\mathbb{E}_t[e^{\int_t^T e^{-\kappa(T-s)}\ln J dq_s}|\mathcal{F}_t] = \exp[\int_t^T e^{-\gamma e^{-\kappa(T-s)}+\gamma e^{-2\kappa(T-s)}}\omega ds - \omega(T-t)]. \tag{4.18}$$

Therefore substituting (4.18) into (4.17) the explicit forward spot price equation is derived to be,

$$\begin{aligned} &F(t,T) = e^{-\lambda\int_t^T \sigma e^{-\kappa(T-s)}ds} \\ &\times G(T)(\frac{S(t)}{G(t)})e^{-\kappa(T-t)}e^{\frac{\sigma^2}{4\kappa}[1-e^{-2\kappa(T-t)}]}\exp[\int_t^T e^{-\gamma e^{-\kappa(T-s)}+\gamma e^{-2\kappa(T-s)}}\omega ds - \omega(T-t)]. \end{aligned} \tag{4.19}$$

Since there is no seasonality in the data of price series we are analysing, then $g(t) = 0$ Also from the definition,

$$G(t) = e^{g(t)} \text{ and } G(T) = e^{g(T)},$$

It follows that

$$G(T) = G(t) = 1 \tag{4.20}$$

Using these values and estimated parameters in table 4.2 we compute the Forward prices for different expiries as shown in Table 4.4.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion.

5.1.1 Results and Discussion

In this dissertation, the mean reverting jump diffusion model for the pricing of energy was developed and used to analyze the price dynamics of electricity. We have shown that, the model developed is flexible enough to incorporate the main characteristics of equilibrium electricity prices; that is mean reversion, volatility and spikes (jumps) as well as seasonality though there were no seasonality observed in the data set analyzed. We have shown also that, there is high degree of volatility and existence of price spikes in the market. From the analysis, the data were found to be appropriate for the stochastic model developed as shown in Figures 4.1 to 4.5 that explain the model relation to the data by showing the main features of the data described by the model. The figures were drawn using Matlab software. The graphs shows that, the prices were mean reverting around the long-term mean price as shown in Figures 4.1, 4.2, 4.9 and 4.10. Together with all those features, the prices dynamics modeled follows non-Gaussian process due to fatty tails shown in Figure 4.8.

As to the output of the model, the simulated price path is similar to the evolution of electricity spot prices as observed in the market, because it covers the price series interval above and below, also the long-term mean price that is considered as the fair price is shared by both real and simulated price series except few jumps which were considered as outliers during simulation as shown in Figure 4.10.The descriptive statistics for both empirical and those from the model are close to each other, showing that the model have almost represented well the price process. With regards to the forward prices, the prices depend much on the market price of risk which is inevitable for incomplete markets. These forward prices are very useful since power markets are one hour/day ahead markets for spot markets.

5.2 Recommendations and Future work.

Affine processes can be applied to different types of processes such as those with stochastic volatility and jumps without sacrificing option pricing tractability. The results of this work can be used by power companies to price the power generated at the market or direct to the final consumers. Therefore it is recommended that;

1. Power Company in Tanzania and East Africa in general should consider the results of this work for proper pricing techniques of electricity within the region.

2. The proposed introduction of energy in East Africa should be implemented to speed up the grid interconnection between the regions for efficient power supply among the East African countries.

3. Application of stochastic models in energy related and financial products is important as their stochastic nature convinces the use of it in modeling.

Unlike other commodity prices, electricity prices show a high degree of persistence in both price levels and returns prices. Therefore in future, next line of empirical research is;

1. Expanding the model to incorporate the intra-day variations caused by different working hours in a day.

2. Applying Markov Chain Monte Carlo method to the process of pricing of electricity to get a better pricing model for reliable price determination at the market.

3. Structural models of demand and supply should be involved to bring insight into the price generating process so as to produce more accurate forecasts.

4. Also we should be able to compute the prices of various electricity derivatives (options) under affine jump-diffusion price processes by exploiting the transform analysis when applicable.

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APPENDIXES

A : Proof of Expected Value in Forward Equation.

In order to evaluate $E_t[e^{\int_t^T e^{-\kappa(T-s)} \ln J_s dq_s}]$, we make the following substitutions.

$$\beta_s = e^{-\kappa(T-s)} \ln J_s \tag{.1}$$

Then we have the following equation

$$E_t[e^{\int_t^T e^{-\kappa(T-s)} \ln J_s dq_s}] = E_t[e^{\int_t^T \beta_s dq_s}] \tag{.2}$$

We first calculate (.2) in the interval $[0, t]$ and then we extend the calculation to the interval $[t, T]$. Let us define L_t such that,

$$\begin{aligned} L_t &= e^{\int_0^t \beta_s dq_s} \\ &= e^{m_t} \end{aligned} \tag{.3}$$

where

$$m_t = \int_0^t \beta_s dq_s \tag{.4}$$

and equivalently

$$dm_t = \beta_t dq_t. \tag{.5}$$

The process described in equation (.5) incorporates jumps and consequently in order to write the SDE followed by L_t , we need to use generalized Ito's Lemma for jumps as given by Etheridge.

$$dL_t = \frac{\partial L_t(m_t-)}{\partial m_t} dm_t - \frac{\partial L_t(m_t)}{\partial m_t}(m_t - m_t-)dq_t + (L_t - L_t-)dq_t. \tag{.6}$$

Notice that, there is no double derivative term in the equation (.6) because our underlying process given by the equation (.5) is a pure jump process without the drift and Wiener process terms as explained by Etheridge and we know that $dq_t \sim dt$.

For the purpose of clarity, the notation m_t- refers to the time index immediately before a jump occurs so that if there is a jump in $\{m_t\}_{t>0}$ it is of magnitude β_t . Mathematically,

$$m_t = m_t - +\beta_t \tag{.7}$$

Using the result above and the fact that $\frac{\partial L_t(m_t-)}{\partial m_t} = L_t-$ equation (.3) is rewritten as

$$\begin{aligned} L_t &= e^{m_t+\beta_t} \\ &= L_t - e^{m_t}, \end{aligned} \tag{.8}$$

and back substituting the transformed equation into equation (.6) we get

$$dL_t = L_t - \beta_t dq_t - L_t - (\beta_t)dq_t + (L_t - e^{\beta_t} - L_t-)dq_t \tag{.9}$$

$$dL_t = L_t - (e^{\beta_t} - 1)dq_t \tag{.10}$$

As stated earlier, we integrate the SDE above from $[0, t]$ and then extending the result to the interval $[t, T]$.

$$\int_0^t dL_s = \int_0^t L_s - (e^{\beta_s} - 1)dq_s \tag{.11}$$

$$L_t = L_0 + \int_0^t L_s(e^{\beta_s} - 1)dq_s. \tag{.12}$$

Since $L_0 = 1$ by definition, we get

$$L_t = 1 + \int_0^t L_s(e^{\beta_s} - 1)dq_s. \tag{.13}$$

Taking expectation of the above and using linearity of the expectation operator with the fact that $\mathbb{E}_0[dq_t] = \omega dt$, gives,

$$\mathbb{E}_0[L_t] = 1 + \int_0^t \mathbb{E}_0[L_s](\mathbb{E}_0[e^{\beta_s}] - 1)\omega ds \tag{.14}$$

where ω the intensity of the Poisson process as has been defined in (3.4).

Defining $\mathbb{E}_0[L_t] = \eta_t$, we can rewrite (.14) as

$$\eta_t = 1 + \int_0^t \eta_s(\mathbb{E}_0[e^{\beta_s}] - 1)\omega ds \tag{.15}$$

which can be differentiated with respect to t to obtain

$$\frac{d\eta_t}{dt} = \eta_t(\mathbb{E}_0[e^{\beta_t}] - 1)\omega. \tag{.16}$$

Again we integrate over the interval $[0, t]$ to get

$$\int_0^t d\eta_s = \int_0^t \eta_s(\mathbb{E}_0[e^{\beta_t}] - 1)\omega ds. \tag{.17}$$

upon integrating and noting that $\eta_0 = L_0 = 1$ we get,

$$\ln \eta_t - \ln \eta_0 = \int_0^t (\mathbb{E}_0[e^{\beta_t}] - 1)\omega ds$$

$$\tag{.18}$$

$$\eta_t = e^{\int_0^t (\mathbb{E}_0[e^{\beta_t}]-1)\omega ds}.$$

Then replacing the definitions of η_t in equation (.18) and L_t in equation (.4) we obtain,

$$\mathbb{E}_0[e^{\int_0^t \beta_s dq_s}] = e^{\int_0^t (\mathbb{E}_0[e^{\beta_t}]-1)\omega ds}. \tag{.19}$$

As stated earlier, the result above in equation (.19) can easily be extended for the interval $[t, T]$ by using the fact that for any integrable function on the interval $[0, T]$

$$\textstyle\int_0^T f(x)dx = \int_t^T f(x)dx - \int_0^t f(x)dx$$

where $t \in [0, T]$.

Hence, this simple extension yields the required result

$$\mathbb{E}_t[e^{\int_t^T \beta_s dq_s}] = e^{\int_t^T (\mathbb{E}_0[e^{\beta_t}]-1)\omega ds}. \tag{.20}$$

Therefore, the log-normal expectation in the forward price equation (4.17) simplifies to evaluating the integral given in the (.20). So, in order to evaluate the integral, the following definition is applied to simplify the process.

$$g(s) = e^{-\kappa(T-s)}. \tag{.21}$$

Then

$$\mathbb{E}_0[e^{\beta_s}] = \mathbb{E}_0[e^{g(s)\ln J_s}]$$

(.22)

$$= \mathbb{E}_0[e^{g(s)\Phi_s}] = e^{-\gamma g(s)+\gamma g^2(s)}.$$

Where γ is the mean of the jump process as explained in equation (3.4). It follows that,

$$\mathbb{E}_t[e^{\int_t^T \beta_s dq_s}] = e^{\int_t^T (e^{-\gamma g(s)+\gamma g^2(s)}-1)\omega ds},$$

(.23)

$$= \exp[\int_t^T e^{-\gamma e^{-\kappa(T-s)}+\gamma e^{-2\kappa(T-s)}}\omega ds - \omega(T-t)].$$