Fluid Mechanics and Kalman Filtering for Computational Market Dynamics

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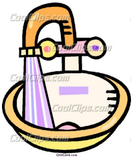
Faucet and sink principle

Question: How much is the liquidity of the currency that is in circulation?

If we know, how much of the commodity (e.g., gold) comes in and goes out, a very simple conservation law holds:

$$\frac{d(\text{amount of gold})}{dt} = (\text{gold mined} - \text{gold destroyed}) \qquad (1)$$



Integral form of conservation law.

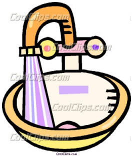
$$\int_V \left(\frac{\partial C}{\partial t} - \mathbf{v} \cdot \nabla C \right) dV = \text{source of C} - \text{sink of C} \qquad (2)$$

Form (2) is also applicable to the multidimensional case. $\mathbf{v} \cdot \nabla C$ states that C is conserved by the flow, apart from internal changes in each control volume represented by sources and sinks.

Only the changing proportion of a conserved variable can be measured and modeled with a conservation law!

In practice we usually split a volume of interest into many ”‘small”’ control volumes and apply (2) to each control volume separately.

In CFD, the conserved quantity can be e.g., mass, energy or momentum.

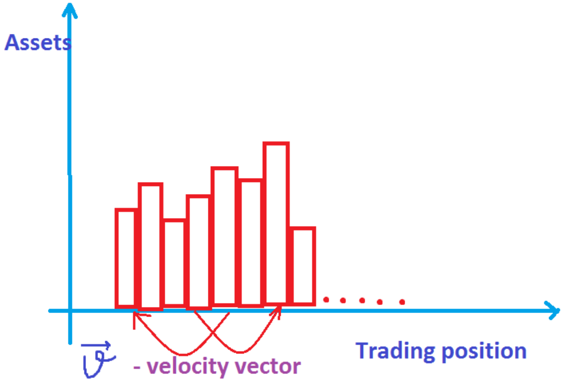


Financial Markets vs. Fluid Mechanics

For Financial Market trading topology is not directly related to physical topology (e.g., a procedure of buying and selling through the net), in contrast to Fluid Mechanics.

Considering ∞-dimensional Fluid Mechanics, we don't restrict ourselves with any topology.

Discretizing the domain



Laws of Market dynamics

1. Conservation of assets

$$\int m(price)\,\mathrm{d}price = const$$

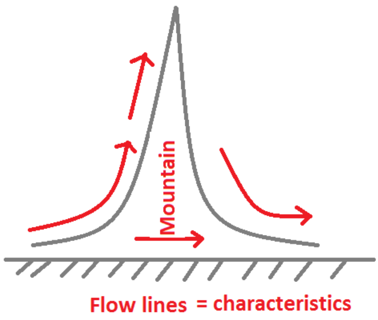
2. Continuity of change

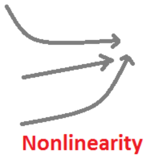
Continuous time distribution implies that each individual transaction is small (microscopic). Hence the global dynamics will be continuous.

Potential flow

In potential flow some quantity (e.g., energy or volume or something else) is conserved along the flow lines (characteristics). Potential might be caused by the pressure, for instance. If flow lines for preasure converge, preasure density increases.

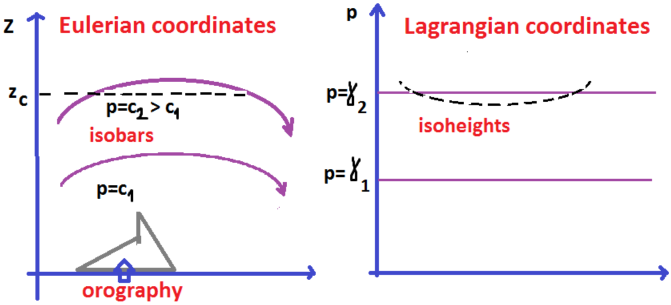
$$\frac{dE}{dt} = 0, \text{ on a flow line} \tag{3}$$



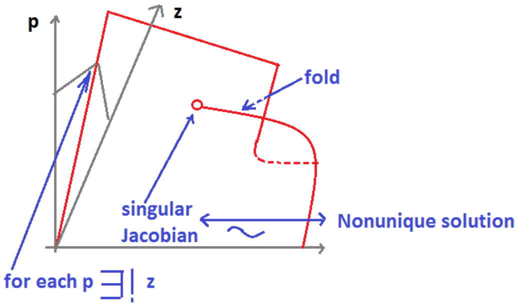


Hydrostatic flow

In hydrostatic flow preasure decreases with height and pressure curves play the role of characteristics. Kalman dynamics can be seen as analogous to semi-Lagrangian advection in CFD - just in Hilbert space!

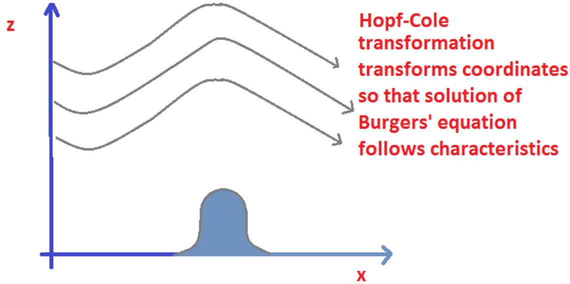


Developed nonlinearity

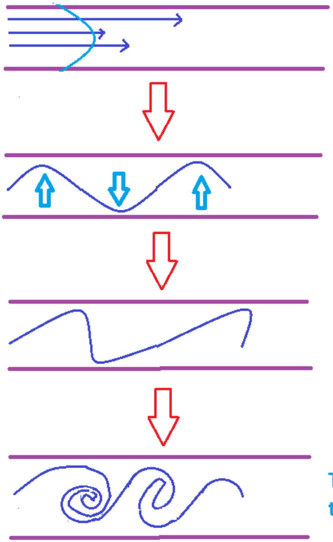


Matching a proper coordinate transformation

The problem is to find a coordinate transformation (normalization),that makes your time series appear wave-like.



Cascade from laminar to turbulent flow.



Laminar flow: velocity trajectories don't intersect, hence, Lagrangian approach can be employed.

Velocity profile starts oscillating (e.g., due to friction).

Advection causes it become asymmetric.

Turbulent flow. Velocity trajectories intersect.

Thank you!