Multi-agent stochastic simulation for the electricity spot market price

No author

1 Introduction

After the Great Recession of 2008-2009, many mathematical and econometric models used in economy have received a lot of criticism, since they were not able to predict the emergence of the asset bubble in the U. S. housing market. As a result of this, econometricians have increasingly turned towards seeking explanations to what happened in the psychological element in market traders' actions. This has repeatedly brought up the idea of emotions that influence human economic behavior. These emotions are also known as *animal spirits* and were originally introduced by John Maynard Keynes in his 1936 book [9].

Not only global economic upheavals display such behavior. A different type of extreme event can be observed in deregulated electricity spot markets which are known to be one of the most volatile financial markets. This distinctive phenomenon is the appearance of price spikes, i.e. sudden price changes to values up to dozens of times higher within only an hour, and again falling back to the previous level within a couple of hours or days. After each spike, market specialists are able to find a reason that caused it in hindsight. But few of those reasons are reliable predictors for future spikes [2]. Nor has any econometric model shown any skill in forecasting those sudden price changes.

In the current study, we investigate the possible origin of price spikes in animal spirits that rule the behavior of all traders. Our modelling attempt of electricity spot markets is not directly based on any notion of intelligent – if emotional – agents. Instead, we have sought to equip classical Ornstein-Uhlenbeck type mean-reverting econometric models with new non-linear terms that emulate the impact of a distinct animal spirit each. Although the motivation for these models is not grounded in the psychology of intelligent agents, they display very similar behavior to that of models based on intelligent agents – or the real price history of electricity on the Nord Pool Spot market, the largest electricity spot market in the world.

This modelling approach is not limited to only spot markets for electricity, where prominent spikes can partly be understood as a consequence of the non-storability of electricity. After suitable normalizing transformations, identical models simulate accurately the behavior of other commodity markets. We have conducted such a study also on the oil spot market for Brent crude. In both cases, the free parameters in the terms of the models are calibrated with a Bayesian Maximum Likelihood principle from the real time series. After this step, the resulting simulated price series reproduces the distribution of the real price series almost exactly up to the sixth statistical moment.

2 Theoretical framework

In the paper's long version, in this section we discuss the theories applied later on in our multi-agent ensemble model. That includes review of electricity spot markets character [16, 23, 12, 6, 21, 18], the trading psychology driven by animal spirits [9, 13, 5, 10, 14, 1, 19, 11, 4, 3] and basic idea standing behind the Capasso-Morale-type population dynamics model [15].

3 Multi-agent simulation for electricity spot market

3.1 Data

The data used for this study is the daily system price from the Nord Pool electricity spot market covering a horizon of over 10 years from January 1999 to February 2009. However, for model and simulation calibration we do not take the original prices, but its detrended and deseasonalized version. More motivation and details are presented in the long version of this contribution [23, 12, 20, 17].

3.2 Simulation and results

In this study we propose to represent individual spot price traders as an ensemble. Price realizations of all of them are described with a system of stochastic differential equations (*Lagrangian representation*). To reflect reality, we set the ensemble size to 300, because currently the Nord Pool market has approximately 330 participants. In particular, each of those differential equations has form (1)

$$dX_t^k = \gamma_t[(X_t^* - X_t^k) + (f(k,\mathbf{X}_t) - X_t^k)]dt + \sigma_t dW_t^k + {}^{+}J_t^k dN_t + {}^{-}J_t^k dN_t, \quad (1)$$

for $k = 1,\ldots,N$, where X_t^k is the price of trader k at time t, X_t^* is the global price reversion level at time t, γ_t is the mean reversion rate at time t, $\mathbf{X}_t$ is the vector of all traders' prices at time t, $f(k,\mathbf{X}_t)$ is a function describing local interaction of trader k with his neighbors (small range of individuals from vector $\mathbf{X}_t$), W_t^k is the Wiener process value for trader k at time t, σ_t is the standard deviation for Wiener increment at time t, ${}^{+}J_t^k$ is the positive jump for trader k at time t, ${}^{-}J_t^k$ is the negative jump for trader k at time t, N_t is the count process for jumps at time t.

In this model we follow the global mean reversion level X_t^* and rate γ_t in a moving fashion with half a year historical horizon (182 days). This feature represents *short-term thinking*, that is one of the main trading biases characterizing market participants. The local interaction $f(k,\mathbf{X}_t)$ is based on following the mean value of neighbors within price range equal to 10% of the total price range, and it stands for the *herding* bias. The jump processes ${}^{+}J$ and ${}^{-}J$ are dependent on current price level at each time t, as we know that electricity spot price is more likely to spike from higher levels than from lower. Therefore, spikes generated by the jumps are reflecting *panic* reaction of traders in the uncertain environment, on both positive and negative side. These emotions originate in *fear and greed*, often felt to be the driving forces behind market movements.

In Fig. 1 we can see the original price and example simulated trajectory (for one out of 300 traders) together with their respective histograms. We can see that the simulation nicely follows the original data both in the long term and appearance of spikes.

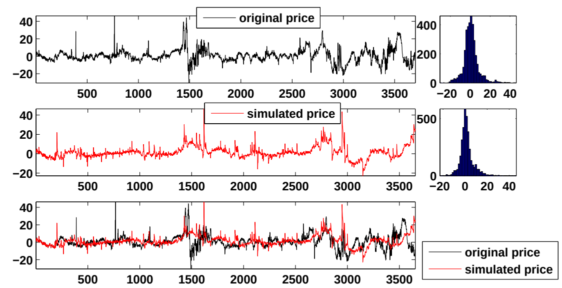

Fig. 1 Ensemble simulation: global reversion to moving mean level with moving rate, and local to neighbors' mean.

The original and simulated histograms are similar. However, we want to quantify the difference as well. Therefore, Table 1 collects comparison of the basic statistics for original pure trading prices and the mean ensemble values.

To complement the whole analysis we employ one more comparison measure, i.e. the probability of the series to exceed specific levels. These probabilities are illustrated in Fig. 2. Clearly, the real data's probabilities fall within the envelope of the whole ensemble. That confirms statistical accuracy and robustness of our approach.

Table 1 Original and ensemble statistical moments: global reversion to moving mean level with moving rate, and local to neighbors' mean.

	Original	Ensemble
Means	0.7286	1.6948
St dev	7.4742	6.1412
Skewness	0.9231	0.9445
Kurtosis	6.9756	6.9135
5th moment	18.8175	22.0969
6th moment	104.9133	150.4392
7th moment	423.4505	832.3929

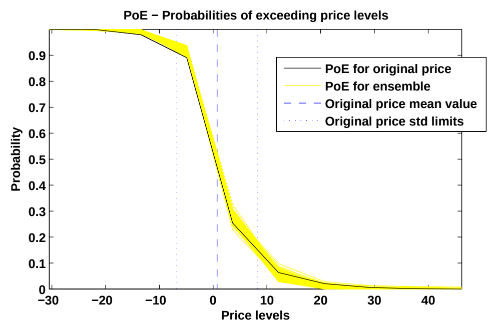

Fig. 2 Original pure price and ensemble probabilities of exceeding specific levels.

4 Conclusions

The multi-agent model proposed in this paper was based on a Capasso-Morale-type population dynamics Lagrangian approach, where movement of each individual is described by a separate stochastic differential equation. However, these particles keep interacting with each other at each time instant, on both local and global basis. We proposed a system that catered for the most common trading biases, i.e. short-term thinking and herding. Also, we included terms representing panic that originates from market uncertainty.

Simulation results presented in our paper prove that our approach reconstructs well many statistical features of the real spot price trading. That was measured by comparing distribution histograms of the original and simulated series, statistical central moments up to the 7th one, as well as a measure showing probability of the prices exceeding specific levels. All these showed remarkable resemblance.

As a future work we suggest improving the multi-agent model by employing more elaborate functions for the local interaction, as well as inclusion of potentials that would represent the market information available to the traders.

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