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Burgers' equation as a model for electricity spot price behavior

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The topic of this Master's thesis was approved
by the faculty council of the Faculty of Technology on

The examiners of the thesis were: Prof. Tuomo Kauranne and Prof. Heikki Haario.
The thesis was supervised by Prof. Tuomo Kauranne.

Lappeenranta, 2011

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Thesis for the Degree of Master of Science in Technology

2011 year

44 pages, 24 figures, 8 tables

Supervisors: Prof. Tuomo Kauranne.
Examiners: Prof. Tuomo Kauranne and Prof. Heikki Haario.

Keywords: Burgers' equation, finite-difference method, Shuman filter, Gaussian filter, NordPool, electricity spot market,pure trading series.

The thesis studies the usability of a one-dimensional fluid model, Burgers' equation to simulating the behavior of electricity spot prices. The price is simulated as a point value of the fluid, and the statistical distributions of both the price and the fluid are compared. We first solve numerically the nonhomogeneous viscid Burgers' equation with periodical boundary conditions by using a finite-difference scheme. After that we study two different solution filtering technique (Shuman filter and Gaussian filter) and then apply each method to the equation. Also we examine how changing parameters of the Burgers' equation affect the distribution of the solution and compare the results with the distribution of pure trading time series of electricity spot market prices as a possible representation of one-dimensional fluid dynamics.

Acknowledgements

I would like to express my special gratitude to Prof. Tuomo Kauranne for his supervising, advices, priceless help, patient and understanding. Also I'm graceful to Prof. Matti Heilio, Prof. Heikki Haario and Matylda Jablonska for their guidance during my work. They gave me force to continue this work.

I would like to say thanks to all my friends. Especially big thanks to Marina Komissarova, Katerina Kurdenkova and Mika-Markus Kepsu. You was with me during this time and did not allow me to give up.

Especial thanks to my parents for their support and love at each step of my life.

Without all of them I could have not completed this work.

Lappeenranta, 2011

Lukyanova Ksenia

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INTRODUCTION

Burgers' equation is a one-dimensional analogue of the Navier-Stokes' equation. It is an important model in fluid mechanics. Particularly, this equation models different kind of physical phenomena, for example, turbulence. Besides, because of its similarity to the Navier-Stokes equation, the solution of Burgers' equation is a first step in the design of new numerical methods for flow problems. In particular, Burgers' equation is used in fluid dynamics teaching and in engineering as a simplified model for turbulence, boundary layer behavior, shock wave formation, and mass transport. It allows us to sidestep the difficulties encountered. In multidimensional fluid dynamics Burgers' equation is a very useful model for numerical experiments and can be solved by using many known numerical methods[6].

In present the work we will consider the initial-boundary value problem for nonhomogeneous Burgers' equation with a specific periodic force-function, initial and boundary conditions. We study a simple numerical method - a finite difference method - and apply this finite difference scheme for solving Burgers' equation. We then conduct several experiments with changing of parameters of the equation to see how it influences the solution. Then we will study solution filtering and apply two different filters to the solution of the Burgers' equation: the Shuman filter and the Gaussian filter and compare the results. In the last part we will try to find out which values of the parameters provide a distribution of the solution value closest to the distribution of pure trading time series of electricity spot market prices as a possible representation of one-dimensional fluid dynamics.

1 The Burgers' equation

1.1 Basics of Burgers equation

In mathematics, partial differential equations (PDE) are a collections of various type of differential equations, i.e., a relation involving an unknown function (or functions) of several independent variables and their partial derivatives with respect to those variables. Partial differential equations are used to formulate, and thus aid the solution of problems involving functions of several variables, such as the propagation of sound or heat, electrostatics, electrodynamics, fluid flow, and elasticity.

One of the major challenges in the field of complex systems is a thorough understanding of the phenomenon of turbulence. A successful theory of turbulence is still lacking which would allow us to predict features of technologically important phenomena like turbulent mixing, turbulent convection, and turbulent combustion on the basis of fundamental fluid dynamical equations. This is due to the fact that already the evolution equation for the simplest fluids, which are the so-called Newtonian incompressible fluids, have to take into account nonlinear as well as nonlocal properties:

$$\frac{\partial}{\partial t}u(x,t) + u(x,t)\nabla u(x,t) = -\nabla p(x,t) + D\Delta u(x,t). \tag{1}$$

Nonlinearity stems from the convective term and the pressure term, whereas nonlocality enters due to the pressure term. In 1939 the Dutch scientist J.M. Burgers (1895-1981) simplified the Navier-Stokes equation (1) by just dropping the pressure term and got a one dimensional nonlinear parabolic PDE of second order:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial t^2} - u\frac{\partial u}{\partial x}\left(\frac{u^2}{2}\right), \tag{2}$$

where $\frac{\partial u}{\partial t}$ is the unsteady term, $D\frac{\partial^2 u}{\partial t^2}$ - the viscous term, $u\frac{\partial u}{\partial x}$ - the convective term, or Burgers' equation can also be written:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}. \tag{3}$$

The solution $u(t, x)$ is a velocity-function of the temporary variable t and the spatial variable x. Subscripts indicate partial differentiation with respect to the given independent variable, and the constant D, which can be interpreted as viscosity, controls the balance between convection and diffusion. Note, that the nonlinear Burgers' equation (2) can be converted into the linear heat equation:

$$\frac{\partial}{\partial t}\psi(x,t) = D\frac{\partial^2 \psi(x,t)}{\partial x^2}. \tag{4}$$

by the Hopf-Cole transformation:

$$\psi(y,t) = \exp -\frac{1}{2D}\int_{\alpha}^{y} u(\xi,t)d\xi, \tag{5}$$

where α is an arbitrary constant [18].

Also note, if $D = 0$, we get the inviscid hyperbolic Burgers' equation:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}, \tag{6}$$

which is a prototype for equations for which the solution can develop discontinuities (shock waves)[24].
In this work we include the force-function F(x,t) in the right-hand side of Burgers' equation and investigate the following equation:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} - vu\frac{\partial u}{\partial x} - F(x,t). \tag{7}$$

The nonhomogeneous periodic function $F(x, t)$ plays the role of a pressure gradient. We use the following type of force-function which is given by

McDonough and Yang[13, 14]

$$F = 105\pi \sum_{j=1}^{K} \left\{ \frac{20A_j^{2-\alpha}\pi}{D} \sin^{19} f_j \cos^2 f_j - \frac{A_j^{2-\alpha}\pi}{D} \sin^{21} f_j - A_j^{1-\alpha} \sin^{21} f_j \cos f_j \right\}$$
$$- \frac{525\pi}{A_j^{2\alpha-1}} \sum_{j=1}^{K} \left\{ \sin^{20} f_j \cos f_j \right\} \sum_{j=1}^{K} \left\{ \sin^{21} f_j \right\}, \quad (8)$$

where A_j is a factor defined by $A_j = e^{0.31j}$,$j = 1, \ldots, K$, and $f_j = A_j\pi(x+t)$.

The value of α is a prescribed constant for each specific nonhomogeneous function. This function has a powerful influence on the form of the solution of Eq. (7).

1.2 The boundary value problem for Burgers' equation.

A boundary value problem is a differential equation together with a set of additional constraints, called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions[21]. To be useful in applications, a boundary value problem should be well posed. This means that given the input to the problem there exists a unique solution, which depends continuously on the input. In the present work we use periodic-type boundary conditions. The meaning of periodic boundary conditions is that the solution to differential equation on the both boundaries of the space interval has the same behavior, i.e. the domain is essentially compact and boundary-less. Define Burgers' equation on $x \in [a, b]$ and $t \in [0, T]$ and let us modify the boundary conditions, which was given in [15], to periodic boundary conditions:

$$u(a,t) = \sum_{j=1}^{K} \frac{5}{A_j^{\alpha}} \sin^{21}\left[A_j \pi (a+t)\right]$$
$$u(b,t) = \sum_{j=1}^{K} \frac{5}{A_j^{\alpha}} \sin^{21}\left[A_j \pi (b+t)\right] \tag{9}$$

and initial condition at time $t = 0$:

$$u(x,0) = \sum_{j=1}^{K} \frac{5}{A_j^{\alpha}} \sin^{21} A_j \pi x. \tag{10}$$

1.3 Burgers' equation in terms of the electricity spot market.

Electricity spot prices typically exhibit periodicity (at annual, weekly and daily horizons), mean reversion, very high volatility and sudden price spikes, all of which can be traced to supply and demand related causes. Burgers' equation can be applied to simulate the behavior of electricity spot prices. In this case we can use the following meaning of parts of the Burgers' equation:

u - the price-function,
Du_{xx} - the spot market tends to reach an equilibrium price,
u_x - the spread between price mean and mode,
uu_x - traders' collective movement towards higher price on the momentum term,
Periodic character of the boundary conditions can describe periodicity of spot prices[19].

2 A finite-difference method for solving Burgers' equation.

2.1 Finite-difference method.

Often, for problems of heat flow, or unsaturated water flow or contaminant transport in soil, there may be no analytic solutions or neat equations describing the result. In such cases, we use numerical methods. Perhaps the simplest of the numerical methods to understand and to program on a computer are finite difference methods (FDM). FDM provides the best combination of speed and accuracy of the solution and ease of implementation of the computational algorithm. This method is based on the approximation of derivatives by finite differences[2]. That is, because the first derivative of function f in point x is, by definition[4],

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(h)}{h}. \tag{11}$$

So, an approximation for the first-order derivative can be taken as:

$$f'(x) \approx \frac{f(x+h) - f(h)}{h}. \tag{12}$$

This is the forward difference equation. Also, we can use a backward difference equation:

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}. \tag{13}$$

And, finally, a central difference equation:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}. \tag{14}$$

For the second-order derivative the central difference equation will be:

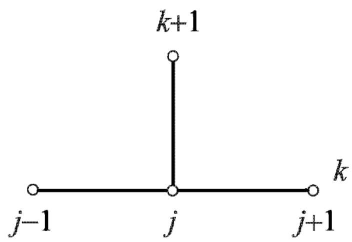
$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}. \tag{15}$$

And the forward difference is:

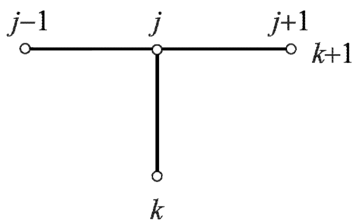
$$f''(x) \approx \frac{f(x+2h) - 2f(x-h) + f(x)}{h^2}. \tag{16}$$

When a particular finite difference equation is chosen, we get a finite difference scheme for the corresponding partial differential equation. The most frequently used are the following patterns of difference schemes:

1. Explicit scheme (use forward difference for first-order derivative and central difference for second-order):



2. Implicit scheme (use backward difference for first-order derivative and central difference for second-order):



The algorithm of the finite difference method is:

1. Determine the nodes of the grid.

2. Replace all derivatives in the equation by approximately equivalent finite differences according to the chosen scheme in each inner node.

3. On the basis of the boundary and initial conditions establish values of the solution at the boundary nodes.

4. Add boundary and initial conditions to difference equations for inner nodes.

5. Convert the system obtained in step 4 into a system of linear equations and solve to get a solution in each node.

Approximation accuracy depends on the chosen scheme of numerical differentiation.

2.2 A finite-difference method for solving Burgers' equation.

We have the following Burgers' equation:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} - vu\frac{\partial u}{\partial x} - F(x,t). \tag{17}$$

where $x \in [a,b]$ and $t \in [0,T]$. We partition the domain in space using a mesh $x_1, ..., x_N$ and in time using a mesh $t_1, ..., t_M$, where N is the number of points on the space interval and M is the number of points on the time interval, including the boundary points. We assume a uniform partition both in space and in time, so the difference between two consecutive space points will be h and between two consecutive time points will be τ. We use the forward difference scheme for first-order derivative and the central scheme for second-order:

$$\frac{\partial u}{\partial t} = \frac{u_n^{m+1} - u_n^m}{\tau}, \tag{18}$$

$$\frac{\partial u}{\partial x} = \frac{u_{n+1}^m - u_{n-1}^m}{2h}, \tag{19}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{n+1}^m - 2u_n^m + u_{n-1}^m}{h^2}, \tag{20}$$

where $h = \frac{b-a}{N-1}$,$\tau = \frac{1}{M-1}$,$n = 1 \dots N$,$m = 1...M$.
Hereby, we get the following system of difference equations:

$$\frac{u_n^{m+1} - u_n^m}{\tau} = D\frac{u_{n+1}^m - 2u_n^m + u_{n-1}^m}{h^2} - vu_n^m \frac{u_{n+1}^m - u_{n-1}^m}{2h} - F_n^m \tag{21}$$

or

$$u_n^{m+1} = u_n^m + \tau D\frac{u_{n+1}^m - 2u_n^m + u_{n-1}^m}{h^2} - \tau vu_n^m \frac{u_{n+1}^m - u_{n-1}^m}{2h} - \tau F_n^m. \tag{22}$$

Then we add initial-boundary conditions and get the following system:

$$u_n^{m+1} = u_n^m + \tau D\frac{u_{n+1}^m - 2u_n^m + u_{n-1}^m}{h^2} - \tau vu_n^m \frac{u_{n+1}^m - u_{n-1}^m}{2h} - \tau F_n^m,$$
$$n = 2, ..., N-1, m = 2, ..., M, \tag{23}$$

$$\begin{aligned} u_n^1 &= \varphi_n, n = 2, ..., N-1, \\ u_1^m &= \mu 1^m, m = 1, ..., M, \\ u_N^m &= \mu 2^m, m = 1, ..., M, \end{aligned} \tag{24}$$

where

$$\varphi_n = \sum_{j=1}^{K} \frac{5}{A_j^{\alpha}} \sin^{21} A_j \pi x_n, \tag{25}$$

$$\mu 1^m = \sum_{j=1}^{K} \frac{5}{A_j^{\alpha}} \sin^{21} A_j \pi (x_1 - t_m),$$

$$\mu 2^m = \sum_{j=1}^{K} \frac{5}{A_j^{\alpha}} \sin^{21} A_j \pi (x_N - t_m), \tag{26}$$

$$\tag{27}$$

$$F_n^m = 105\pi \sum_{j=1}^{K} \left\{ \frac{20 A_j^{2-\alpha}\pi}{D} \sin^{19} f_{jn}^m \cos^2 f_{jn}^m - \frac{A_j^{2-\alpha}\pi}{D} \sin^{21} f_{jn}^m - A_j^{1-\alpha} \sin^{21} f_{jn}^m \cos f_{jn}^m \right\}$$
$$- \frac{525\pi}{A_j^{2\alpha-1}} \sum_{j=1}^{K} \{\sin^{20} f_{jn}^m \cos f_{jn}^m\} \sum_{j=1}^{K} \{\sin^{21} f_{jn}^m\}, \tag{28}$$

3 Applying Burgers' models to electricity spot markets.

3.1 Electricity spot markets.

An electricity market is a system for effecting purchases, through bids to buy; sales, through offers to sell; and short-term trades, generally in the form of financial or obligation swaps. Bids and offers use supply and demand principles to set the price. Long-term trades are contracts similar to power purchase agreements and generally considered private bi-lateral transactions between counterparties[22].

3.1.1 Nord Pool Electricity spot market.

According to [19] liberalization of power market was pioneered in 1982 by Chilie. The reform was based on the idea of separate generation and distribution companies where power was paid for according to a formula based on the cost, a dispatch system with marginal cost pricing and a system of trading power between generators to meet customer contracts. This reform was followed by the reorganization of the British electricity sector in 1990. In 1991 the Parliament of Norway decided to deregulate the market for power trading. Then in 1992 Nord Pool, now known as an electricity market for the Nordic region, was founded. In 1993 Statnett Marked AS (now Nord Pool ASA) was established as an independent company. In 1996 Norway and Sweden established a common electricity market and power exchange. It was a pioneer in power exchange. Then Finland (1998) and Denmark (2000) joined in. The Nordic electric power market features direct trading among players (bilateral trade) and trading via the power exchange, Nord Pool Spot. Players from outside the Nordic region are allowed to participate in this market on equal terms with 'local' exchange members. To participate in the spot (physical) market, called Elspot, a grid connection enabling power to be delivered to or taken from the main grid is required. Additionally, a continuous hour-ahead Elbas market is also operational in Finland, Sweden and eastern Denmark. There are today over 300 market participants from over 10 countries active on Nord Pool. These include generators, suppliers/retailers, traders, large customers and financial institutions. The success of Nord Pool can be explained by several factors. Firstly, the industry structure is very fragmented with over 350 generation companies. Secondly, large amount of hydropower allows storage and flexibility in production. Thirdly, the structure of the network is relatively simple, compared to continental Europe, which facilitates congestion management. Finally, the level of collaboration between system operators, governments and regulators is very high in contrast to the many conflicts of interest between continental European countries[23].

3.1.2 Price formation at Nord Pool Spot.

The primary role of a market price is to establish an equilibrium between supply and demand. This task is especially important in the power markets because of the inpassibility to store electricity and the high costs associated with any supply failure. The Spot market at Nord Pool Spot is an auction based exchange for the trading of prompt physically delivered electricity. It is the central Marketplace for Nordic Electricity[23]. According to [19] the spot price at Nord Pool is a result of a two-sided uniform-price auction for hourly time intervals. Elspot is a market for trading power for physical delivery. It is a day-ahead market. Nord Pool Spot publishes a spot price for each hour of the coming day in order to synthetically balance supply and demand. Every morning Nord Pool participants post their orders to the auction for the coming day. Each order specifies the volume in MWh/h (minimum contract size is 0.1 MWh) that a participant is willing to buy or sell at specific price levels (EUR/MWh). This information is provided electronically via the Internet (Elweb) with a resolution of one hour, i.e. one for each hour of the next day. Such information should contain both price and volume of the bids. To be correct, there are three possible ways of bidding at Elspot. First, hourly bidding consisting of pairs of price and volume for each hour. Second, in block bidding, the bidding price and volume are fixed for a number of consecutive hours. Third, flexible hourly bidding is a fixed price and volume sales bid where the hour of the sale is flexible and determined by the highest (next day) spot price that is above the price indicated by the bid. However, the system used by Nord Pool shares many common features with other power exchanges. By 12 p.m. Nord Pool closes the bidding for the next day and for each hour proceeds to make cumulative supply and demand curves. Since there must be a balance between production and consumption, the system spot price for that particular hour is determined as the price where the supply and demand curves cross. Hence the name market cross or equilibrium point. Trading based on this method is called equilibrium trading, auction trading or simultaneous price setting. If the data does not define an equilibrium point, no transactions will take place for that hour. After having determined the system price for a given hour of the next day's 24-hour period, Nord Pool continues by analyzing for potential bottlenecks (grid congestions) in the power transmission grid that might result from this system price. If no bottlenecks are found, the system price will represent the spot price for the whole Nord Pool area. However, if potential grid congestion may result from the bidding, so-called area spot prices (zonal prices), that are different from the system price, will have to be computed. The idea behind the introduction of area (zonal) prices is to adjust electricity prices within a geographical area in order to favor local trading to such a degree that the limited capacity of the transmission grid is not exceeded[18, 23].

3.2 Characteristics of electricity spot prices.

3.2.1 Seasonality.

According [10, 18] demand and supply of electricity depend on season. Basically, they arise due to changing climate conditions, like temperature and the number of daylight hours. These seasonal fluctuations in demand and supply translate into seasonal behavior of electricity prices, and spot prices in particular. Also the supply side shows seasonal fluctuations in production. In some markets, however, no clear annual seasonality is present and the spot prices behave similarly throughout the year with spikes occurring in all seasons (examples are Spain, Czech Republic, Poland where most of spikes are negative, and Italy). Many statistical observations show us that, for the Nordic countries, a typical behavior of the spot price process is the same with a sinusoid with a linear trend. The sinusoid nearly duplicates the long-term annual fluctuations - high prices in winter time and low prices during the summer. Based on such observations, Pilipovic (1998) advocated the use of the 'sinusoidal' approach for electricity price modeling. Apart from the annual sinusoidal behavior there is a substantial intraday variability. Higher than average prices are observed during the morning and evening peaks, while mid-day and night prices tend to be lower than average. The intra-week variability, related to the business day weekend structure, is also no negligible. The price begins to increase at roughly 6h a.m., as the populace wakes and the workday begins. This price increase continues throughout the day as demand builds, peaking at 16h. Prices begin to fall thereafter as the workday ends and demand shifts to primarily residential usage. Higher prices appear from Tuesdays to Fridays, with the highest spikes occurring at Friday (weekly effects), and around 9 am to 12 am (daily effects). However, prices fall back to normal levels overnight. Cuaresma et al. (2004) report higher prices during weekdays, and intraday patterns and price spikes. The weekday prices are higher than those during the weekends, when major businesses are closed. If we plot realizations of hourly prices over weekly periods, intraday and weekly seasonal patterns - pronounced early morning and late afternoon demand-driven peaks - with the exception of Sundays, where the morning peak is absent, are evident. The modeling of intra-week and intraday seasonalities may be approached analogously to modeling annual fluctuations, i.e. by simply taking a sine function of a one week period, or better a sum of sine functions with distinct periods to recover the non-sinusoidal weekly structure. Alternatively, we may apply the moving average technique, which reduces to calculating the average weekly price profile or just extract the mean or median week. Bhanot (2000), Knittel and Roberts (2005) and Lucia and Schwartz (2002) use piecewise constant functions; Cartea and Figueroa (2005) and Pilipovic (1997) model the seasonal pattern by sinusoidal functions; while Stevenson (2001) uses a wavelet decomposition.

3.2.2 Volatility.

According to [8] in finance, volatility is a measure for variation of price of a financial instrument over time. With deregulation and introduction of competition a new challenge has emerged for power market participants. Extreme price volatility, which can be even two orders of magnitude higher than for other commodities or financial instruments, has forced producers and wholesale consumers to hedge not only against volume risk but also against price movements. Price forecasts have become a fundamental input to an energy company's decision-making and strategy development. This in turn has propelled research in energy price modeling and forecasting. The volatility encountered in electricity markets is exceptional and not comparable with the one observed in other commodity and financial markets. Applying the standard concept of volatility Weron (2005) obtains: for notes and treasury bills less than: 0.5%; stock indices: 1 - 1.5%; commodities like natural gas or crude oil: 1.5 - 4%; very volatile stocks: not more than 4%; and electricity up to 50%. The high volatility is a pattern due to the transmission and storage problems and of course the requirement of the market to set equilibrium prices in real time. Since it is not easy to correct provisional imbalances of supply and demand in the short-term, the price changes are more extreme in electricity markets than other financial or commodity ones. Volatility is often viewed as a negative in that it represents uncertainty and risk. However, it can be good in that if one shorts on the peaks, and buys on the lows one can make money, with greater money coming with greater volatility. Due to the non-storability characteristic of electricity, this is however not possible in the spot market. The use of derivatives or block contracts trade was created to diminish these kind of impacts. Volatility clustering appears quite evident when we consider entire week days, as opposed to only week work days (Monday to Friday) where volatility is more pronounced and also it presents a more spiky behavior. Volatility clustering refers to the observation that large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes. A quantitative manifestation of this fact is that, while returns themselves are uncorrelated, absolute returns or their squares display a positive, significant and slowly decaying autocorrelation function. Observations of this type in financial time series have lead to the use of GARCH models in financial forecasting and derivatives pricing. Hadsell et al. (2004) estimate volatility, study time series properties of spot electricity prices, and examine regional differences and similarities, using a sample for the NYMEX market, and a TARCH specification. They conclude there is significant price volatility regardless of region, time differences or stage of deregulation. Knittel and Roberts (2005) investigate the behaviour of California's restructured electricity prices using jump diffusion models and exponential GARCH. They have reported seasons, regular intraday pattern, weekday/weekend cycle, time varying and volatility clustering, mean reversion, and also jumps from every 20 to 33 hours[10].

3.2.3 Jumps and spikes.

One of the most pronounced features of electricity markets are the abrupt and generally unanticipated extreme changes in the spot prices known as jumps or spikes. Within a very short period of time, the system price can increase substantially and then drop back to the previous level (see Fig 12 - the Nord Pool system spot prices are shown at an daily time resolution[11]).

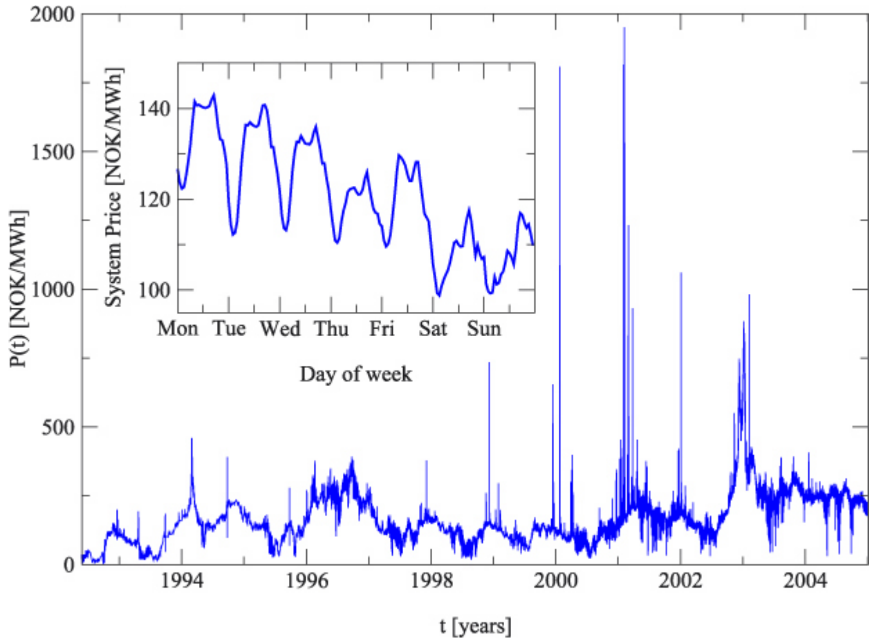


Figure 1: The Nord Pool system spot price from May, 1992 till the end of 2004. Several (seasonal, weekly, and daily) periodicities can be observed in this data set below the spiky randomness. The inset shows the price variations over a (randomly chosen) weekly period.

In obedience to [11] these temporary price escalations accounts for a large part of the total variation of changes in spot prices and firms that are not prepared to manage the risk arising from price spikes can see their earnings for the whole year evaporate in a few hours. The spike intensity is also non-homogeneous in time. The spikes are especially notorious during on-peak hours, i.e. around 09:00 and 18:00 on business days, and during high consumption periods: winter in Scandinavia, summer in mid-western U.S., etc.

For example, in Nord Pool high price spikes occur mostly on winter, when the Nord Pool is surrounded by snow, and since it is highly dependent on hydroelectric power (Weron, 2005). The years of 2003 and 2005 were very important in terms of jump behavior in Europe due to extreme weather conditions. As the time horizon increases and the data are aggregated the spikes are less and less apparent. For weekly or monthly averages, the effects of price spikes are usually neutralized in the data. It is not uncommon that prices from one day to the next or even within just a few hours can increase tenfold. The 'spiky' nature of spot prices is the effect of non-storability of electricity. Electricity to be delivered at a specific hour cannot be substituted for electricity available shortly after or before. As currently there is no efficient technology (at a reasonable price) for storing vast amounts of power, it has to be consumed at the same time as it is produced. Hence, extreme load fluctuations - caused by severe weather conditions (demand sided shocks) often in combination with generation outages or transmission failures (supply side shocks) - can lead to price spikes. The spikes are normally quite short-lived, and as soon as the weather phenomenon or outage is over, prices fall back to a normal level (Geman and Roncoroni, 2006; Seifert and Uhrig-Homburg (2006); among others). Reasons for single, positive or negative, jumps, followed by mean-reverting prices, can be manifold. Power plant or supply line outages can lead to short or long price impacts, depending on the severity and length of the outage. Poisson processes are an easy way to model this jump behavior. Unexpected strong changes in weather can cause price spikes. Spikes are jump patterns which show an initial (positive or negative) jump followed by a reverse directed jump on the next day. Extreme weather situations can also result in very volatile and jumpy price periods due to a high load level. These translate into clusters of jumps in a short time period which can be positive, negative or mixed. There are, however, markets where practically no spikes are present. For instance, in Poland, since the inception of the day-ahead competitive wholesale electricity market in July 2000, no price spikes have been observed! The prices typically range between 80 and 140 PLN/MWh. Even the annual seasonality is not according the data[18]. Anyway, modeling jumps in electricity prices is important to explain observed market data and to account for future price risks. But considering the complex jump patterns identified in electricity markets, the question comes up which jump model can be used.

3.2.4 Mean-reversion.

According to [10, 18] energy spot prices are in general regarded to be mean reverting or anti-persistent. The speed of mean reversion, however, depends on several factors, including the commodity being analyzed and the delivery provisions associated with the commodity. In electricity markets, it is common to observe sudden price spikes with very fast mean reversion to the previous price levels. In natural gas markets, the mean reversion rate is considerably slower, but the volatilities for longer-dated contracts are usually lower than the volatilities for the shorter-dated ones. In oil markets, the mean reversion rate is thought to be longer term, and it can take months, or even years, for prices to revert to their mean. Changes in demand push up electricity prices and increase the economic motivation of expensive suppliers to enter the market. So, it is rational to anticipate the evolution of electricity prices to exhibit mean reversion. Alternatively, it is also natural to say that mean reversion is pronounced in the dynamics of electricity prices, because equilibrium prices are highly influenced from the weather through shifts in demand. The evolution of weather is a mean reverting and cyclical process, thus the tendency it has to go back to its mean level will influence the demand and consequently equilibrium prices. The volatility of electricity prices is mostly affected by the presence of sudden and large variations which, typically, last for one day: upward jumps in the price level are usually followed by downward jumps of almost the same size that revert the price to its "normal" level. Price behavior in the Nord Pool market appears to follow a kind of mean-reverting model with jumps. The mean for which they revert can even change through the years, but prices still come back to the mean level for that specific period.

3.3 Burgers' equation and electricity spot markets.

In this section we study statistical features of given data of pure trading series (PTS) and compare it with simulated data by using Burgers' equation model.

3.3.1 Analysis of pure trading series data

In previous study [8] pure trading series was defined as the price series with removed trend and seasonality. The pure price is a general systematic non-linear component that changes over time and does not repeat or at least does not repeat within the time range captured by the data. The original data used in this study is from Nordic electricity market Nord Pool. The data consists of 3702 daily observations of the NP electricity spot prices. Fig. 2 is the plot of electricity prices of the Nord Pool.

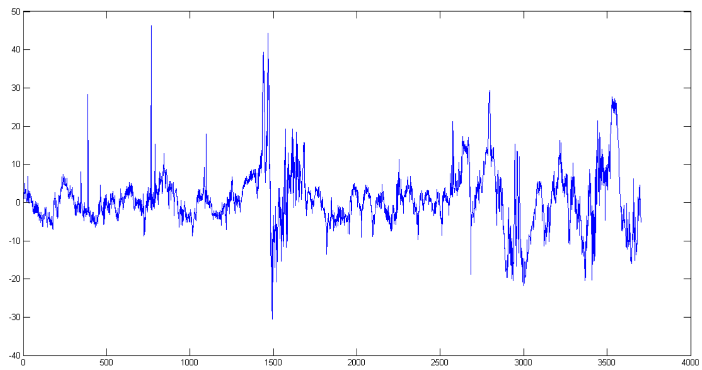


Figure 2: Nord Pool spot prices.

Then we check the type of distributions that pure trading data have. Fig. 3 shows the normalized histogram of our data against theoretical normal probability curve (i.e. PDF).

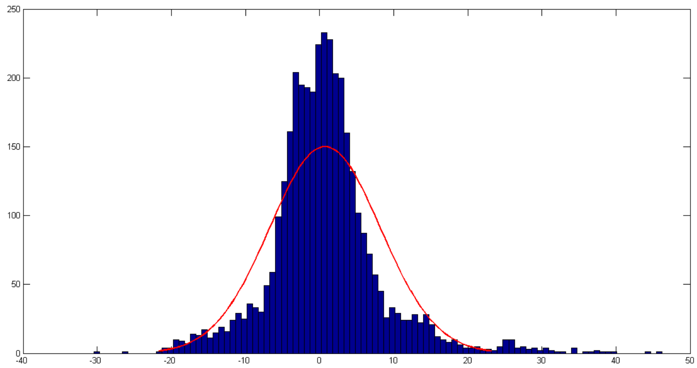


Figure 3: Normalized histogram for NP electricity spot prices.

Table 1 shows the main statistical features of the PTR data. Including the mean value, variance, standard deviation, kurtosis and skewness coefficients.

Table 1: Basics statistics for pure trading series data.

Count	mean	var	std	kurtosis	skewness
3702	0.7286	55.8642	7.4742	6.9756	0.9231

3.3.2 Simulated results by using Burgers' equation model.

In this section we study results obtained by using Burgers' equation as a possible model for simulation of spot prices. In Figures 4-8 we can see the general solution of Burgers' equation $u(x, t)$ on the interval $x \in [0, 1]$ and $t \in [0, 1]$, N=600,step on time-axis $\tau = 0.1$ and D=0.1, v=0.09.

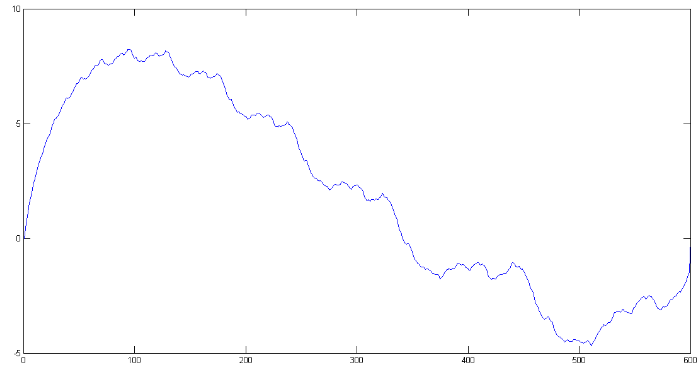


Figure 4: Solution of Burgers' equation in t=0.

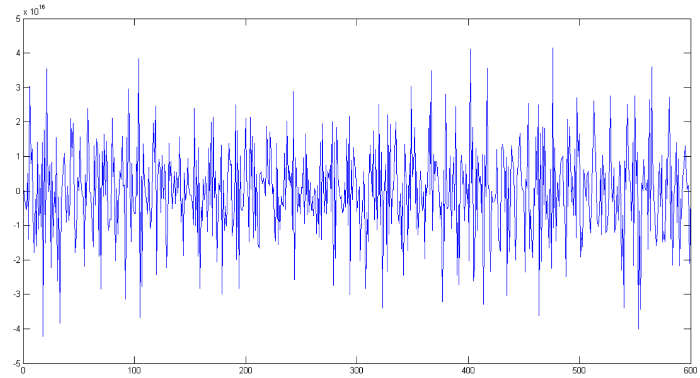


Figure 5: Solution of Burgers' equation in t=0.25.

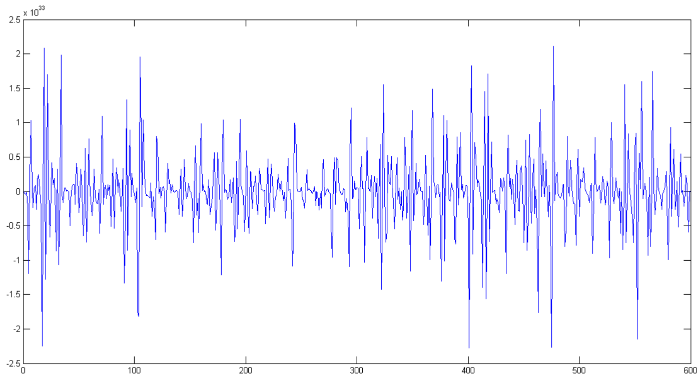


Figure 6: Solution of Burgers' equation in t=0.5.

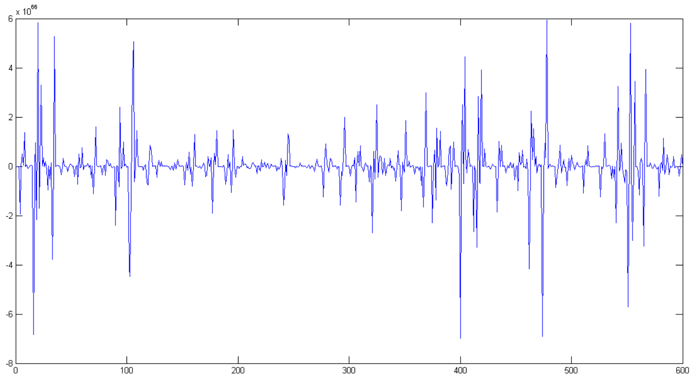


Figure 7: Solution of Burgers' equation in t=0.75.

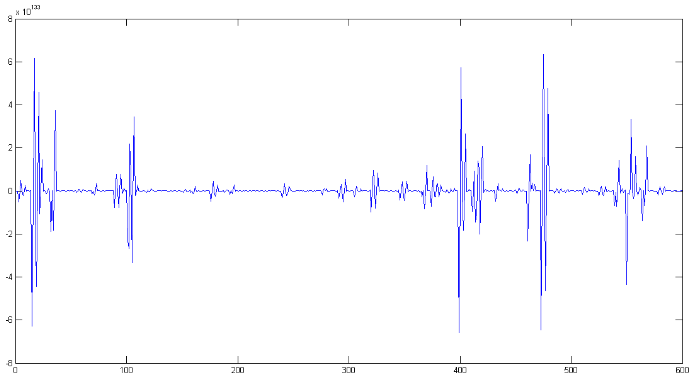


Figure 8: Solution of Burgers' equation in t=1.0.

In the next figure we can see how the solution changes by the influence of diffusion coefficient D with a constant momentum coefficient v=0.09. Blue line corresponds to the diffusion coefficient D=0.1, red is D=0.05 and green is D=0.9.

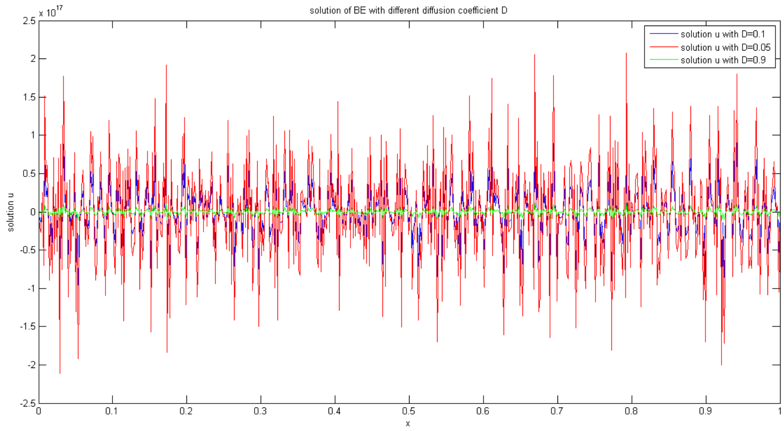


Figure 9: Solution of Burgers' equation at time t=0.25 with different diffusion coefficients D.

In Figure 10 we will show how the momentum coefficient v influences the solution of Burgers' equation. Diffusion coefficient is kept at D=0.07. Blue line corresponds to the diffusion coefficient v=0.05, red is v=2 and green is v=1.

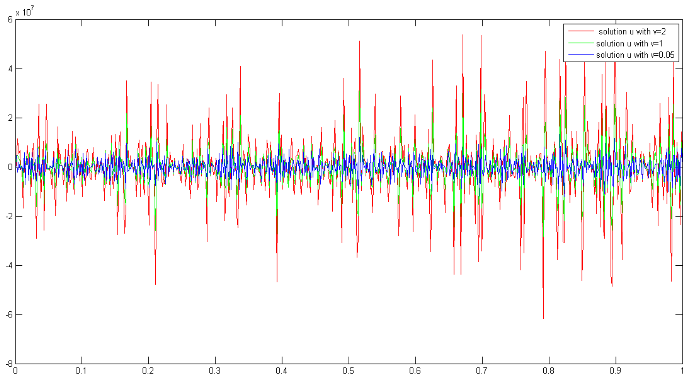


Figure 10: Solution of Burgers' equation for three different values of v at time t=0.25.

In a Figure 11 influence of three different combinations of the coefficients D and v is shown. Time t=0.25.

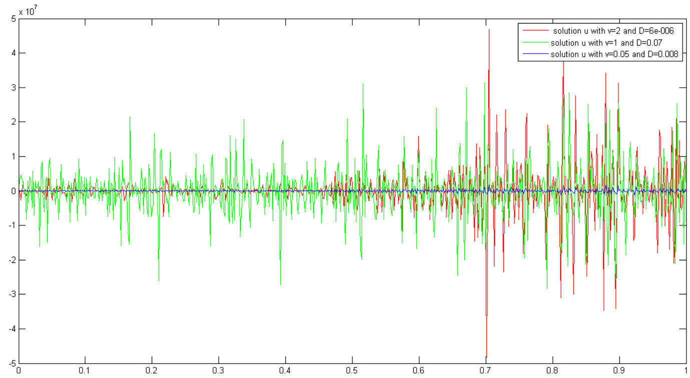


Figure 11: Solution of Burgers' equation with three different combinations of momentum coefficients.

The next step is to plot a histogram of the distribution of Burgers' equation solution. In Fig. 12 we see a comparing of normalized histograms of PTS and Burgers' equation solution with coefficients D=0.1 and v=0.1 against CDF of normal distribution. Table 2 shows basic statics for Burgers' equation solution with comparing PTS for this case. Note: all data in following experiments is for normalized solution u unless otherwise specified.

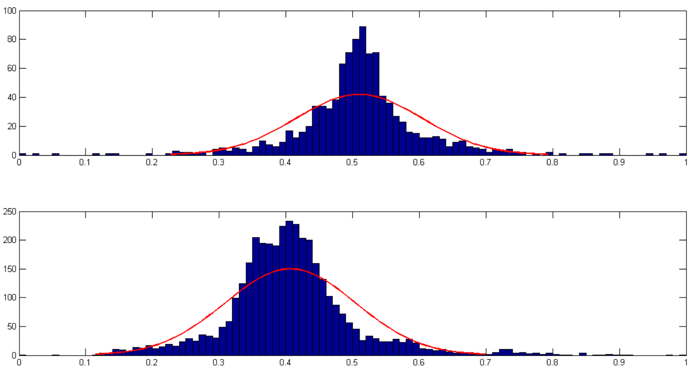


Figure 12: Normalized histograms against CDF of normal probability. Top plot for Burgers' equation solution and lower for PTS data.

Table 2: Basic statistics for Burgers' equation solution and pure trading series data of Nord Pool. Coefficients D=0.1, v=0.1

moment	Burgers' equation solution	Pure trading series	normalized PTS
mean	0.5099	0.7286	0.4067
var	0.0089	55.8642	0.0095
std	0.0943	7.4742	0.0974
kurtosis	8.1639	6.9756	6.9756
skewness	-0.0624	0.9231	0.9231

Next two figures show us how distribution changes when diffusion coefficient increases (On the Fig.13 D=0.5) and decreases (On Fig.14 D=0.005) and coefficient v=0.1 for all histograms. Table 3 and 4 show main statistical moment for both cases respectively.

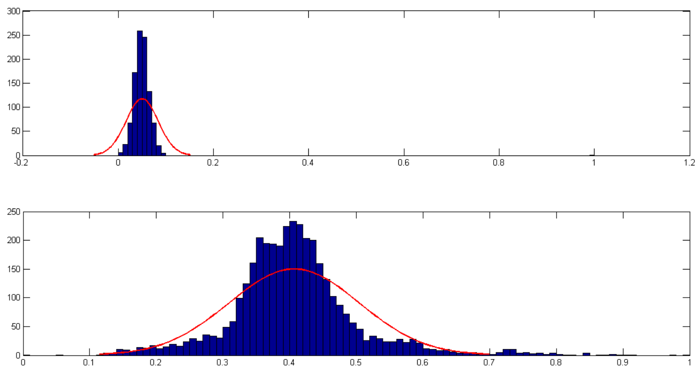


Figure 13: Normalized histograms against CDF of normal probability. Top plot for Burgers' equation solution and lower for PTS data.

Table 3: Basic statistics for Burgers' equation solution and pure trading series data of Nord Pool. Coefficients D=0.5, v=0.1

moment	Burgers' equation solution	Pure trading series	normalized PTS
mean	0.0501	0.7286	0.4067
var	0.0011	55.8642	0.0095
std	0.0337	7.4742	0.0974
kurtosis	633.9491	6.9756	6.9756
skewness	-22.4502	0.9231	0.9231

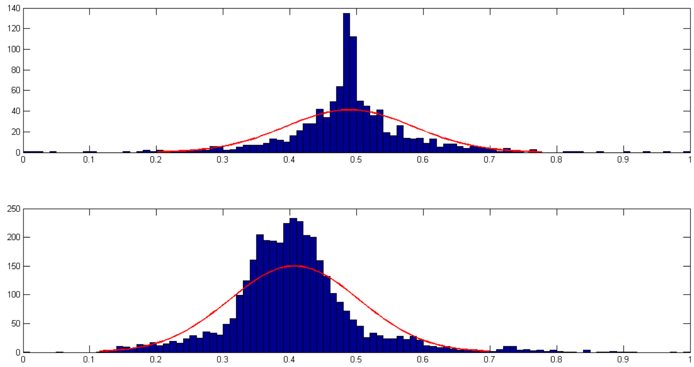


Figure 14: Normalized histograms against CDF of normal probability. Top plot for Burgers' equation solution and lower for PTS data.

Table 4: Basic statistics for Burgers' equation solution and pure trading series data of Nord Pool. Coefficients D=0.005, v=0.1

moment	Burgers' equation solution	Pure trading series	normalized PTS
mean	0.4894	0.7286	0.4067
var	0.0092	55.8642	0.0095
std	0.0960	7.4742	0.0974
kurtosis	8.4259	6.9756	6.9756
skewness	-0.0529	0.9231	0.9231

Next two figures show how coefficient v influences on the distribution with the same value D=0.1 (On the Fig.15 v=0.5 and on the Fig.16 v=0.03). Table 5 and 6 show statistics for both cases.

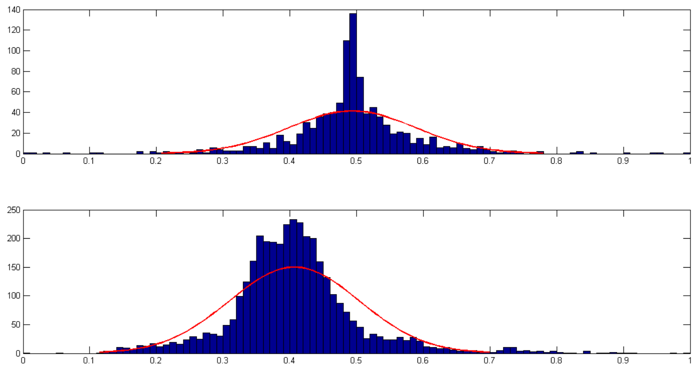


Figure 15: Normalized histograms against CDF of normal probability. Top plot for Burgers' equation solution and lower for PTS data.

Table 5: Basic statistics for Burgers' equation solution and pure trading series data of Nord Pool. Coefficients D=0.1, v=0.5

moment	Burgers' equation solution	Pure trading series	normalized PTS
mean	0.4936	0.7286	0.4067
var	0.0091	55.8642	0.0095
std	0.0953	7.4742	0.0974
kurtosis	8.4189	6.9756	6.9756
skewness	-0.0541	0.9231	0.9231

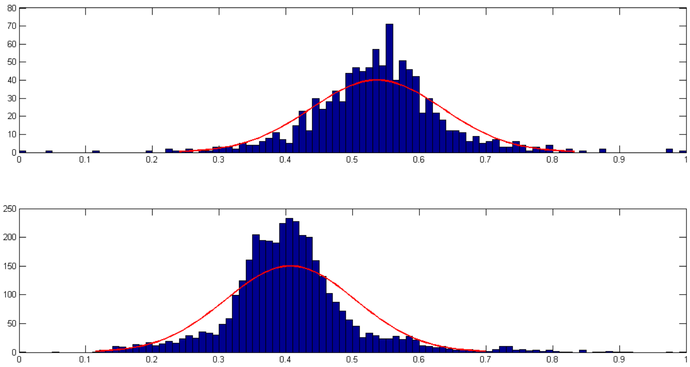


Figure 16: Normalized histograms against CDF of normal probability. Top plot for Burgers' equation solution and lower for PTS data.

Table 6: Basic statistics for Burgers' equation solution and pure trading series data of Nord Pool. Coefficients D=0.1, v=0.03

moment	Burgers' equation solution	Pure trading series	normalized PTS
mean	0.5364	0.7286	0.4067
var	0.0097	55.8642	0.0095
std	0.0986	7.4742	0.0974
kurtosis	6.3745	6.9756	6.9756
skewness	-0.0747	0.9231	0.9231

3.3.3 Filtering technique for solution of Burgers equation.

The solution filtering technique (SFT) is a numerical method for solving, for example, turbulence problems. It uses the filtering idea of the classical large-eddy simulation. In this part we will apply two different filtering techniques for the solution of Burgers equation. The solution is filtered spatially in each time step and then filtered result is applied to the next time step. The first filter is the Shuman filter. It uses dependent variable values in a small neighborhood of grid points in "physical" space to remove the effects of aliasing[13, 14]. The formula for this filter is:

$$u_i = \frac{u_{i+1} + \beta u_i + u_{i-1}}{2 + \beta}, i = 2, ..., N-1. \tag{29}$$

where β is filtering parameter. In Figure 17-18 result of the applying the Shuman filter is shown. Coefficients D=0.1, v=0.03, N=1000 and $\beta = 10$.

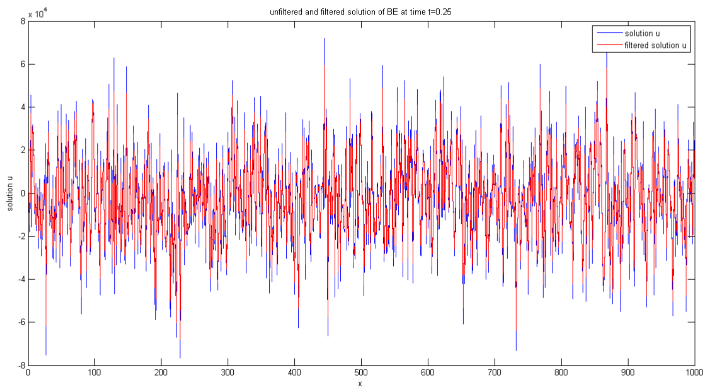


Figure 17: Results of applying the Shuman filter to the solution at t=0.25 .

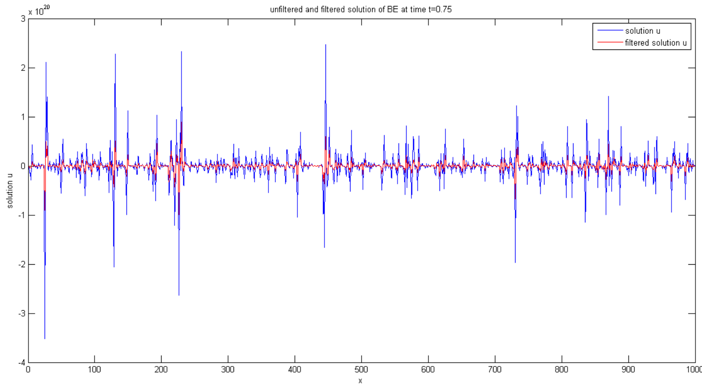


Figure 18: Results of applying the Shuman filter to the solution at t=0.75 .

The next filter which we use is the Gaussian filter. Basically, this filter is used in large-eddy simulation of turbulent flows, and has the following form:

$$u_i = \frac{u_i + \sum_{j=1}^{m} u_{i-j} \exp^{-\lambda j^2} + u_{i+j} \exp^{-\lambda j^2}}{1 + \exp^{-\lambda j^2}}, \tag{30}$$

where λ is a constant parameter. The Gaussian filter cannot be applied to grid points nearest to the boundary if the value of the filter m is larger than 1 [15]. In Figure 19-20 you can see the result of using the Gaussian filter for $\lambda = 2$,m=8 and coefficient of Burgers' equation is D=0.1 and v=0.03.

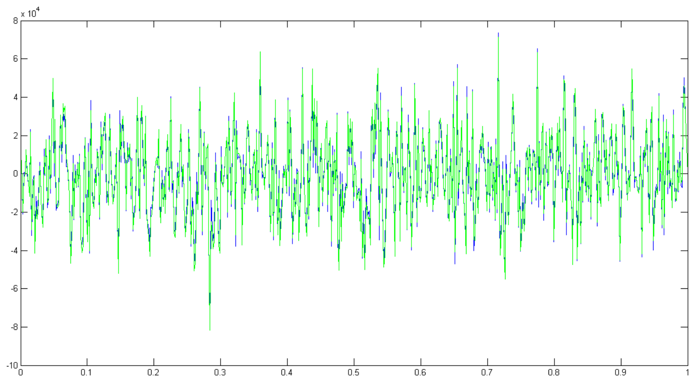


Figure 19: Unfiltered and filtered solution by the Gaussian filter at t=0.25.

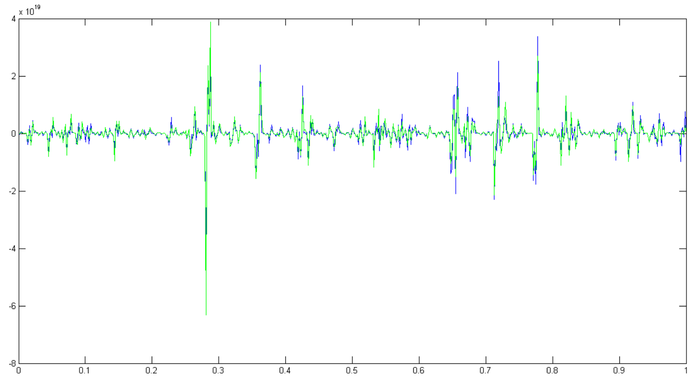


Figure 20: Unfiltered and filtered solution by the Gaussian filter at t=0.75.

Let's compare the results which are given by the Shuman and Gaussian filters. Value of parameters: D=0.1, v=0.03, β for Shuman filter is 10 and λ for Gaussian filter is 1. We can see that the Shuman filter works better.

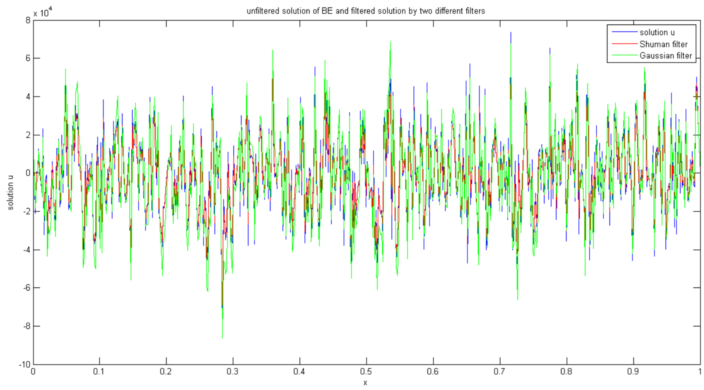


Figure 21: Shuman and Gaussian filters at t=0.25.

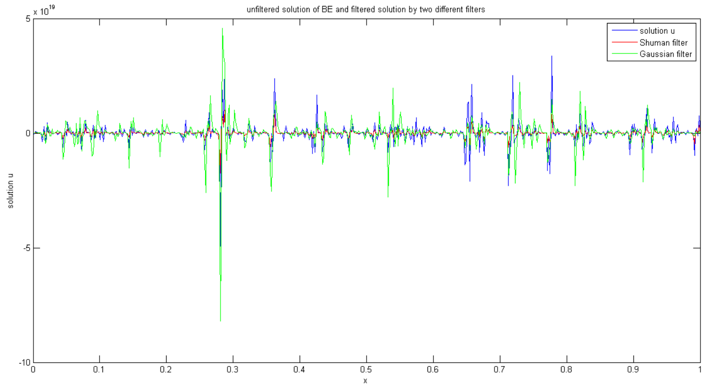


Figure 22: Shuman and Gaussian filters at t=0.75.

As last step we compute a several experiments to show how distribution of Burgers' equation solution changes when we change the value of parameters for both filters, β and λ for Shuman and Gaussian filters respectively. We can see results on the Fig. 21 (for Shuman filter) and Fig. 22 (for Gaussian filter) with comparing PTS data distribution. In Tables 7,8 statistical results for our experimental data is shown.

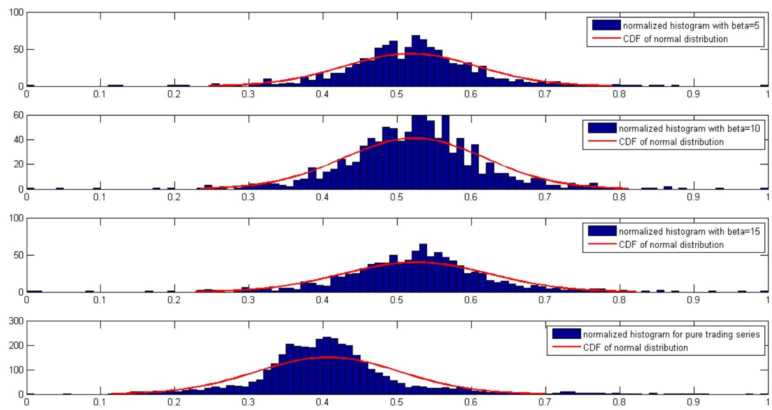


Figure 23: Normalized histograms of filtered solution with different filter parameter β against CDF of normal probability. Last plot - histogram for PTS data.

Table 7: Basic statistics for filtered solution of Burgers' equation by Shuman filter with different parameters and pure trading series data of Nord Pool.

moment	$\beta = 10$	$\beta = 15$	$\beta = 5$	Pure trading series	normalized PTS
mean	0.5230	0.5152	0.5170	0.7286	0.4067
var	0.0093	0.0098	0.0082	55.8642	0.0095
std	0.0963	0.0990	0.0907	7.4742	0.0974
kurtosis	6.3342	6.3164	6.4790	6.9756	6.9756
skewness	-0.0720	-0.0794	-0.0273	0.9231	0.9231

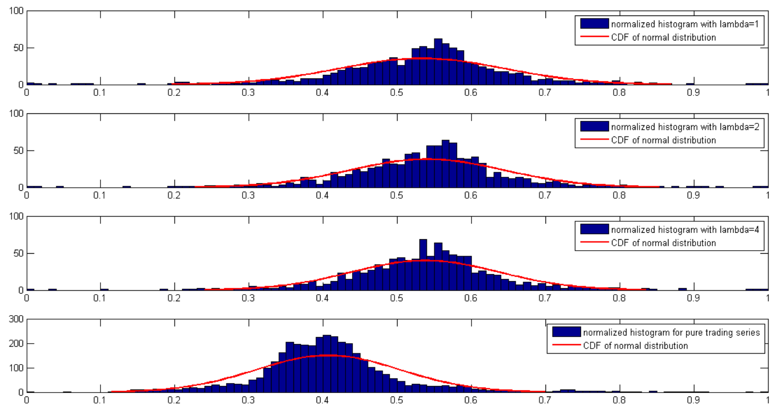


Figure 24: Normalized histograms of filtered solution with different filter parameter λ against CDF of normal probability. Last plot - histogram for PTS data.

Table 8: Basic statistics for filtered solution of Burgers' equation by Shuman filter with different parameters and pure trading series data of Nord Pool.

moment	$\lambda = 2$	$\lambda = 4$	$\lambda = 1$	Pure trading series	normalized PTS
mean	0.5400	0.5378	0.5330	0.7286	0.4067
var	0.0109	0.0099	0.0126	55.8642	0.0095
std	0.1045	0.0993	0.1122	7.4742	0.0974
kurtosis	6.2332	6.383	6.2919	6.9756	6.9756
skewness	-0.2127	-0.0946	-0.4819	0.9231	0.9231

4 Conclusion.

In this thesis we have studied one of the most important models in fluid dynamics namely - Burgers' equation. We have set a periodical force-function and boundary conditions for this equation. Then we have considered the corresponding initial-boundary value problem and used a finite-difference method for solving it. In the third part of this work we have considered Nord Pool as an example of general features of electricity spot markets such as seasonality,volatility,mean-reversion, spikes and jumps. We have used pure trading series data from Nord Pool - 3702 daily observations of electricity spot prices and analyzed its basic statistical characteristics, such as distribution and statistical moments (mean,kurtosis,skewness, etc.). The last part of this work presents simulation results by using Burgers' equation model as a possible model for electricity spot price behavior. We have displayed the general solution at a few different time moments. Then we have studied how changing of diffusion and momentum coefficients D and v, respectively, influences the behavior of the solution. We have noticed that increasing the coefficient D makes the solution smoother by removing sharp "jumps" in the solution. Completely different changes result with changing the coefficient v. The solution becomes smoother, without "jumps" with a smaller value of coefficient. The next step of our study was to investigate statistical characteristics of Burgers' equation in terms of financial markets and compare it with data for pure trading series from Nord Pool. The last step was applying a solution filtering technique for Burgers' equation. We have consider two different filters (Shuman and Gaussian), then computed several experiments with changing of the parameters for these filters. Afterwards, we have compared pure trading series data and Burgers' equation results for filtered solutions.

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