Animal Spirits in Population Spatial Dynamics

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Abstract. Collective behavior of herding animals displays a balance between conservative cohesive forces and "‘innovation"’, or moving towards a goal, as described by Couzin in 2005 (2005). We have given these forces a quantitative mathematical form that is amenable to numerical simulation. The simulations described herein reproduce the phenomena that Couzin observed and indicate that a 5 per cent critical mass is sufficient to pull the whole population along, but smaller innovative teams fail to attract a substantial following. The resulting non-linear dynamic equations have been applied also to modeling of financial market dynamics, where they are seen to produce financial catastrophes by internal population dynamics alone, without any need for external forcing. The equations can thereby also be interpreted as a model of John Maynard Keynes' Animal Spirits (1936) that are often evoked to describe market psychology.

1 Introduction

Animal behaviour in schools, swarms, herds, etc. have always been puzzling even for biologists, who study their habits on a daily basis. For a long time scientists were convinced that direction of, for example, a flock of birds cruising the sky is driven by a single leader in front. But Couzin et al. (2005) showed through numerical simulations that it has to be at least 5% of the total population heading in a specific direction to pull the whole group behind them. That same fact was verified two years later empirically during a big experiment in Cologne by Krause and Dyer (see Mob mentality, 2009). There, a group of 200 people was told to move freely around a large space, 400 by 230 feet, though without communicating with each other and just staying close to their neighbors. After some time the groups tended to move in two concentric circles rotating in opposite direction. Then a small subgroup of all was told to head in a specific direction and it appeared that still 2.5% did not influence the whole population movement, but 5% did interfere the circular motion, causing the main group follow in the same direction.

This very idea has inspired a recent study in financial market modeling by Jabłońska (2011) and Jabłońska and Kauranne (2011), where a one-dimensional model of population dynamics was used to simulate the behavior of an ensemble of traders in electricity spot markets. Their results have shown that if a sufficiently big subgroup of the whole population was bidding far enough from the mean level, the others would follow that direction and skyrocket the price to the level a dozen times higher than the mean.

As the aforementioned model shows interesting properties, the aim of this work is to implement and analyze the same model in two dimensions for simulating dynamics of a group of individuals. The model is formed as a system of coupled stochastic differential equations, each representing one individual in the population. As such, most of the individuals have no intelligence of where they are heading. They only follow interactions within the whole group. However, a small subgroup is given a deterministic movement path, and the simulations verify what has to be the size of that escaping group to pull the other population members with them.

This article is structured as follows. Section 2 presents the family of population dynamics models being the background of the final model. Section 3 describes in detail the components of the final model and their physical and psychological interpretation. In Section 4, numerical simulations are presented for different model settings. Finally, Section 5 concludes and gives ideas for future work.

2 Population spatial dynamics

The aim of this study is to verify whether a specific population dynamics model can reproduce the natural fact that 5% of a population can pull the whole group towards a specific direction. This work presents an extended implementation of a model proposed by Jabłońska (2011) and Jabłońska and Kauranne (2011), which was then successfully used for simulation of electricity spot price behaviour. The model is based on the Capasso-Bianchi system of stochastic differential equations in a general form (1), used for modelling animal population dynamics by Morale et al. (2005) or price herding by Bianchi et al. (2003) and Capasso et al. (2005). The movement of each particle k in the total population of N individuals is based on the location of each individual with respect to the whole population $f(X_t^k)$, as well as on its local interaction with the closest neighbors $h(k, \mathbf{X}_t)$.

$$dX_N^k(t) = [f(X_t^k) + h(k, \mathbf{X}_t)]dt + \sigma dW^k(t), \quad \text{for } k = 1, \dots, N. \tag{1}$$

Jabłońska (2011) and Jabłońska and Kauranne (2011) extended this model with a Burgers'-type momentum component which catered for momentum in financial markets, and implemented it for one-dimensional price dynamics of an ensemble of traders. The referred model is presented in detail in Section 3 in Equations (2)-(4).

The following study presents a two-dimensional version of this model which, with suitable parameter values, can be used for simulating behaviour of large populations of individuals.

3 Model formulation

This section presents in detail the model proposed in this study, as well as numerical simulations with different model parameter values. This work is based on the Capasso-Morale approach mentioned in Section 2. Each individual in the population is followed separately; together they form a system of coupled stochastic differential equations. The structure of each equation is the same and it is related to the main idea of an Ornstein-Uhlenbeck mean reverting processes. However, a single constant mean reversion level is replaced by three individual components, each standing for a specific type of force acting on the population as a whole, and on its individuals separately.

The main components of the proposed model are:

Global mean: the whole population is expected to oscillate around its (moving) center of mass; this is related to the aggregation forces proposed by Morale et al. (2005). This component stands for the herding phenomenon, that is the willingness of the individuals to stay within a bigger group.

Momentum: in particular, the momentum effect should occur when a sufficiently big subgroup of the whole population has significantly different behavior (external information) that deviates from the total population mean. This has been noticed in studies by Couzin et al. (2005).

Local interaction: each individual in the population can perceive its neighbors up to a limited extent, which seems natural especially for big populations. Therefore, each population member will follow the furthest neighbor within a range that caters for the closest $p\%$ of the whole population. This will allow the emergence of a proper repulsion force and avoid overcrowding in any point in space. Also, individuals are deemed to follow the farthest units of their neighborhood, thinking that those have some distinct information in the 'big picture' and, therefore, are far for a good reason.

Randomness: each individual's move includes a Wiener increment to allow randomness in the system.

Hence, the model is defined as Equation (2)

$$d\mathbf{X}_t^k = [\gamma(\mathbf{X}_t^* - \mathbf{X}_t^k) + \theta(h(k, \mathbf{X}_t) - \mathbf{X}_t^k) + \xi(g(k, \mathbf{X}_t) - \mathbf{X}_t^k)]dt + \sigma_t d\mathbf{W}_t^k \quad (2)$$

where

$$h(k, \mathbf{X}_t) = \mathrm{M}(\mathbf{X}_t) \cdot [\mathrm{E}(\mathbf{X}_t) - \mathrm{M}(\mathbf{X}_t)] \quad (3)$$

having $\mathrm{M}(X)$ stand for the mode of a random variable X and $\mathrm{E}(X)$ being a classical expected value. Also,

$$g(k, \mathbf{X}_t) = \max_{k \in I}\{\mathbf{X}_t^k - \mathbf{X}_t\}, \text{ where } I = \{k|_{X^k \in N_{p\%}^k}\} \quad (4)$$

where $N_{p\%}^k$ means the neighborhood of the k-th individual formed by the closest $p\%$ of the population. $\mathbf{X}_t^*$ stands for the mean of the whole population at time t, and parameters γ, θ and ξ are the forces with which each of the interactions takes place. In the original model by Jabłońska (2011) and Jabłońska and Kauranne (2011) these forces are allowed to vary in time, but in this work they are kept constant for simplicity. Also, this work extends previous implementation to the two-dimensional case.

4 Numerical simulations

4.1 Model Parameters

As mentioned before, some of the model parameters are fixed for simplicity and, as there is no real life data available for calibration, they are chosen deterministically. The values are $\gamma = 0.05$ for the global mass center rate, $\theta = 10^{-5}$ for the momentum component and $\xi = 0.9$ for the strength of the local interaction. The percent for the range of the local interaction is chosen as 5% of the total population. With the aforementioned parameter values, the model simulation will follow the movement of a total population of $N = 200$ individuals. Most of them will move only through model dynamics,. Their initial locations generated from a two-dimensional uniform distribution $U^2(-2h, 2h)$, where $h = 0.05$ is also used as a grid step factor for finding the mode of the population at every time step.

4.2 Simulation results

First, the simulation demonstrates the aggregative power of the model, which is apparent even when $\gamma = 0$. The individuals, originally uniformly distributed, quickly cluster into a few compact groups which then follow their own dynamics as presented in Figure 1. As a small subgroup (5%) of individuals moves in a deterministic way, one of the aggregated groups joins them, and follows the deterministic path. Whenever any two groups get close enough to one another, they may merge. In the plots, D denotes the distance between the centers of mass of the subgroup and the remaining population.

In the next simulation the size of the subpopulation, also called the *escape group*, is kept at the same 5% level. However, the global center of mass attraction rate is set to $\gamma = 0.05$, and the escape group is now moving along a circle. As presented in Figure 2, the main stream of the population can easily follow the escape group, keeping the distance between their mass centers at a low and not very volatile level. That is confirmed on the distance plot in Figure 3.

Simulations have shown that any escape size bigger than 5% produces similar results, always keeping both groups close and well aggregated. However, if this value is decreased, the main stream of the population starts having trouble to keep close to the escape group, not even holding the distance constant. This is evident for the escape size of 4%, as depicted in Figures 4 and 5 for the path shapes and the distance plot, respectively. Apparently, the main population gets off the escape group course as the individuals try to cut the way short and catch the subgroup, but even that barely slows down the increase of the distance between the mass centers. Also, the 4% fails to attract even a single member of the main stream to their neighborhood, as opposite to the case from Figure 1.

To verify the threshold at which the main stream of the population has the probability to remain merged with the escape group equal to 1 we run the simulations multiple times for different population percent size with parameter `popperc = [0.04:0.0005:0.05]`, and verify how often the subgroup detaches

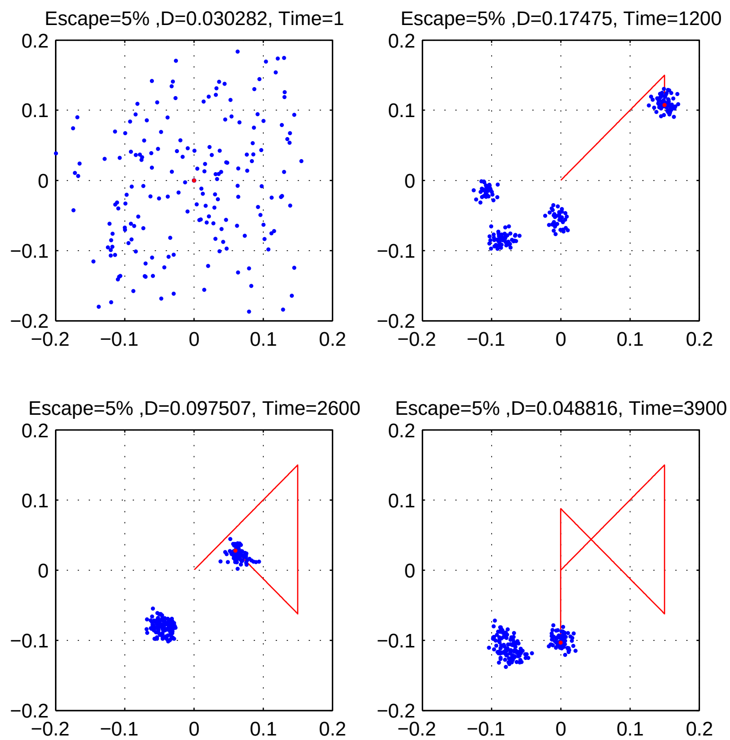

Fig. 1. Progress of particle movement with 5% of the population moving in a deterministic way and non-active center of mass attraction.

from the whole population. For each subpopulation size the simulation is repeated 30 times. Then the distance between the groups is measured. Figure 6 presents the results, showing that the value of `popperc = 0.0485` is the threshold from which onwards the main stream will always remain in contact with the escape group.

The multiple runs allow us to make a rough approximation of probability of the main stream to remain merged with the escape group with respect to different subgroup sizes. As depicted in Figure 7, the probability is dramatically low for 4%, then stays moderate for most of the range, and reaches almost 1 at 4.8%, and 1 at 4.85%.

Finally, once again the global mass center attraction parameter γ is set to zero, to verify what percent size would suffice to hold the whole population compact when following the escape group. It appears, that there are two clear thresholds. At the level of 9% the probability of the whole population remaining as one group increases significantly from an average below 0.2 to an average of

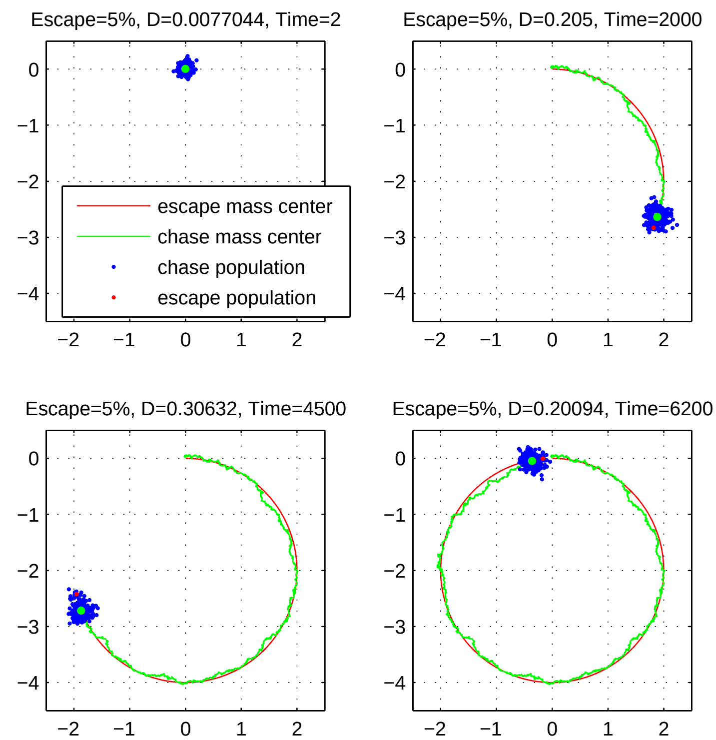

Fig. 2. Progress of particle movement with 5% of the population moving in a deterministic way and active center of mass attraction.

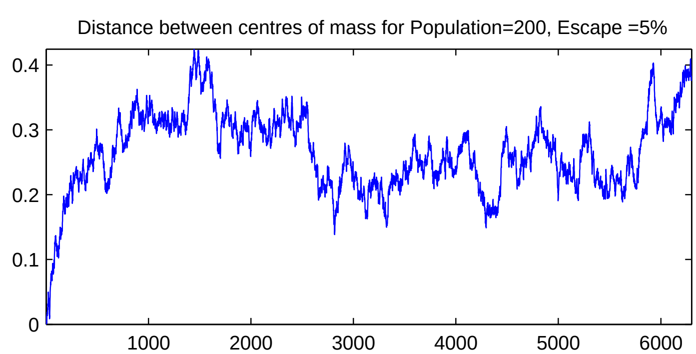

Fig. 3. Distance between the centers of mass of the escape group and the main stream with 5% of the population moving in a deterministic way and active center of mass attraction.

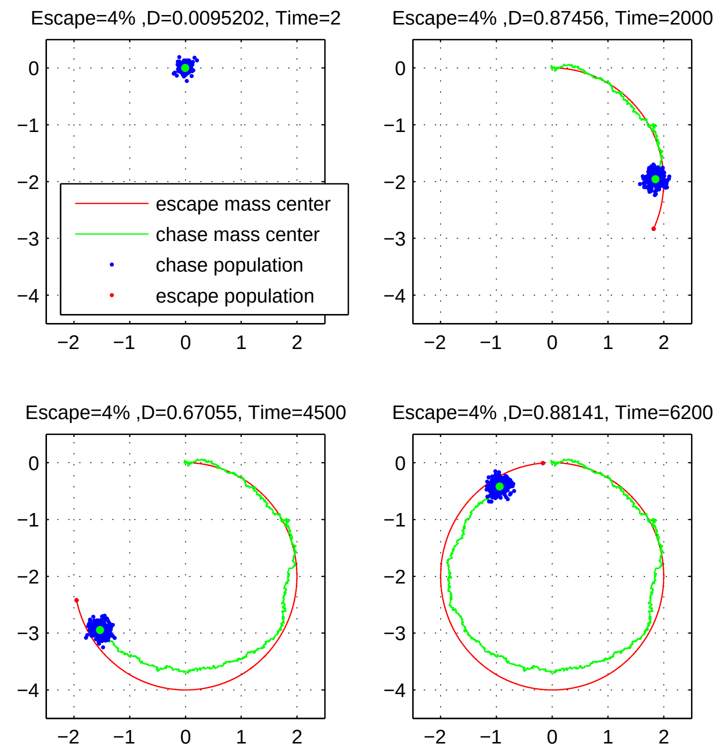

Fig. 4. Progress of particle movement with 4% of the population moving in a deterministic way and active center of mass attraction.

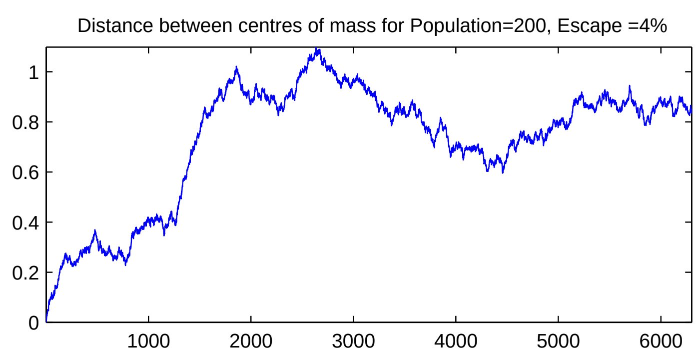

Fig. 5. Distance between the centers of mass of the escape group and the main stream with 4% of the population moving in a deterministic way and active center of mass attraction.

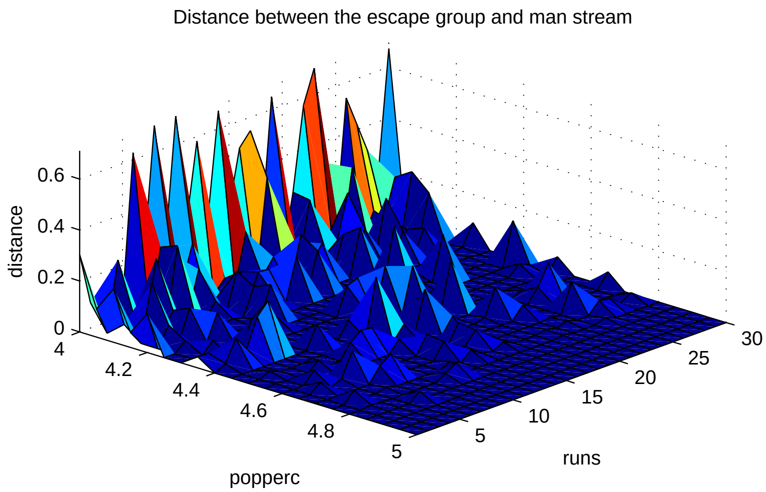

Fig. 6. Distance between the the escape group and the main stream with multiple runs for different sizes of the escape group.

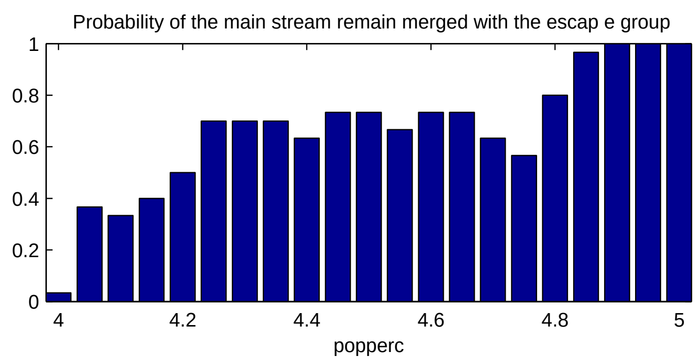

Fig. 7. Probability of the main stream to remain merged with the escape group with respect to different subgroup sizes, with active center of mass attraction.

over 0.7, as presented in Figure 8. However, it is the subgroup size of 9.75% that guarantees the population remain merged with theescape group with probability 1. Otherwise, as presented in Figure 9 for the case of 8%, most of the the main stream of the population may loose interest in the subgroup after some time, and continue its movement only with respect to its own internal dynamics. However, some of the main stream particles stay attracted to the escape group, which is not the case in Figure 4 for the 4% subpopulation, even with the center of mass attraction active in the latter case.

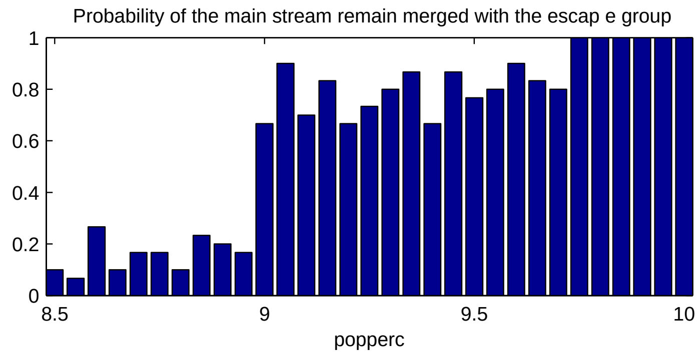

Fig. 8. Probability of the main stream to remain merged with the escape group with respect to different subgroup sizes, without the center of mass attraction.

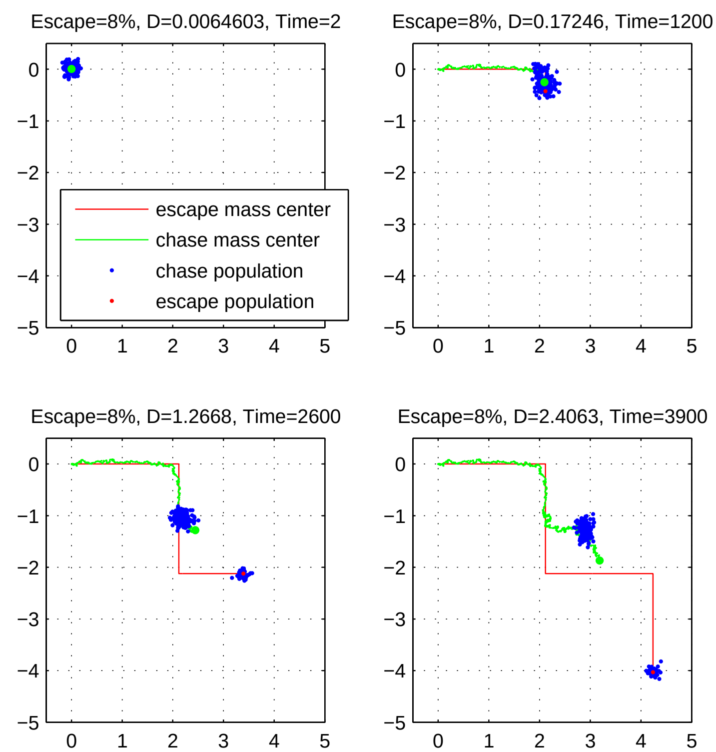

Fig. 9. Progress of particle movement with 8% of the population moving in a deterministic way and non-active center of mass attraction.

5 Conclusions

This study presented a two dimensional implementation of a population dynamics model proposed by Jabłońska (2011) and Jabłońska and Kauranne (2011). This approach stemming from a combination of fluid dynamics and animal spatial dynamics was used previously in modelling financial times eries. In this work it was used to analyze behavior of individuals on a two-dimensional plane, when a specific subgroup of the whole population heads in a deterministically set direction, while the remaining part follows only its internal dynamics.

The results have shown that which particular model setting the value of 5% for the size of the escaping subgroup is a minimum necessary to pull the rest of the population in the same direction and let it stay close to the subgroup. Any smaller size may influence the movement of the whole population, but is not sufficient to attract the individuals to stay constantly close to the escape. This leads to conclusion that, as the model is able to reproduce the natural behaviour of biological organisms (Mob mentality, 2009), then it may be appropriate to be used as a basis for modelling animal behaviour in more complicated settings.

One suggestion for future work is to analyze the population movement when closed in a bounded space, or having obstacles on the way. Moreover, it would be interesting to see the model's performance in a prey-predator setting, that is when the total population would contain two main subpopulations: preys and predators, whose aggregation and repulsion forces would be defined through their biological interactions.

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