Mathematical Epidemiology, Exercise 2

The 1988 PDV seal epidemic

In 1988, the seals in northern Europe were infected with the phocine distemper virus (PDV) and a rapidly progressing epidemic ensued. One data set describing the epidemic, the cumulative number of dead seals found from the eastern coast of England, is given in the table (first column is time, second column is the data).

t	R
0.0000000e+000	0.0000000e+000
1.0000000e+000	6.0000000e+000
2.0000000e+000	1.3000000e+001
3.0000000e+000	4.7000000e+001
4.0000000e+000	2.4300000e+002
5.0000000e+000	3.1500000e+002
6.0000000e+000	4.0700000e+002
7.0000000e+000	7.7400000e+002
8.0000000e+000	9.8900000e+002
9.0000000e+000	1.0700000e+003
1.0000000e+001	1.1490000e+003
1.1000000e+001	1.1590000e+003
1.2000000e+001	1.1800000e+003
1.3000000e+001	1.2010000e+003
1.4000000e+001	1.2300000e+003
1.5000000e+001	1.2370000e+003
1.6000000e+001	1.2470000e+003
1.7000000e+001	1.2520000e+003

A simple SIR compartment model is used to explain the progress of the epidemic. In SIR, the epidemic is divided into three stages (compartments): susceptible S, infected I and removed (dead) R. In addition, a compartment is added for the healed individuals H. Susceptible (potentially infected) individuals are in contact with the infected individuals and catch the infection. The infected individuals can either die or recover and become immune to PDV. The SIR model is written as an ODE system as

$$\dot{S} = -\alpha SI \quad (1)$$
$$\dot{I} = \alpha SI - \beta I \quad (2)$$
$$\dot{R} = (1-\nu)\beta I \quad (3)$$
$$\dot{H} = \nu\beta I \quad (4)$$

where α is the contact rate, β the removal rate and ν the survival rate. The unknown parameters θ in the model are the rates and the initial number of infected individuals: $\theta = [\alpha, \beta, \nu, I(0)]$. The task is to estimate θ from observations for R.

Vary each of the parameter values within normal distribution around the estimated value. How much uncertainty does each of the parameters contribute to the uncertainty of the entire model?