1 Tuberculosis treatment model

The population is divided into four compartments, namely individuals susceptible to tuberculosis (S), exposed individuals (E), infectious individuals (I) and treated individuals (T).

- Susceptible and treated individuals enter the exposed compartment at rates $\beta_1 I$ and $\beta_2 I$, respectively, where $N = E + I + S + T$.
- Exposed individuals progress to the infectious compartment at the rave ν. All newborns are susceptible, and all individuals die at rate $d > 0$. Thus, the core of the model is an SEI model using standard incidence.
- The treatment rates are r_1 for exposed individuals and r_2 for infectious individuals. However, only fraction q of the treatments of infectious individuals are successful. Unsuccessfully treated infectious individuals re-enter the exposed compartment.

Tasks:

- Draw the compartmental diagram of disease dynamics.
- Write down the model equations.
- Derive R_0.

2 Dengue fever: host-vector model

The host population is divided into three compartments, namely individuals susceptible to dengue (S), infectious individuals (I) and recovered individuals (R). The vector population is divided into two compartments, the no-infected (M) and infected (V) individuals.

- Susceptible individuals get infected with rate $\beta_s V$ after contacts with infected vector individuals. Newborns, entering the population at a constant rate b become immediately susceptible.
- Infectious individuals recover at rate r.
- Recovered individuals can remain immune at rate μ. Otherwise, they re-enter the susceptible group.
- New vector individuals enter the population at a constant rate c. The non-infected vector population becomes infected at rate $\beta_m I$ after feeding on the infected hosts.
- All the hosts die at rate b and the vector individuals die at rate c.

Tasks:

- Draw the compartmental diagram of disease dynamics.
- Write down the model equations.
- Derive R_0.

3 HIV/AIDS model: population level

The population is divided into five compartments, namely susceptible individuals (S), infected individuals in the primary (high viral load) HIV stage (Y), infected individuals in the incubation stage of HIV (I_1), the pre-AIDS (high viral load) stage of HIV (I_2), and the treated individuals with full blown AIDS (A).

- Susceptible individuals enter the sexually active population at a rate bN. They become HIV-infected at rate λX. The force of infection is given by

$$\lambda = \frac{c_1\beta_1 Y + c_2\beta_2 I_1 + c_3\beta_3 I_2 + c_4\beta_4 A}{N}$$

- β_1 denotes the per partnership transmission probability of individuals in the Y class.
- β_2 denotes the per partnership transmission probability of individuals in the I_1 class.
- β_3 denotes the per partnership transmission probability of individuals in the I_2 class.
- β_4 denotes the per partnership transmission probability of individuals in the A class.
- c_1, c_2, c_3, c_4 are the average number of new sexual partners acquired per unit time.
- Infected individuals in the primary stage progress to the incubation stage at the rate σ.
- Infected individuals in the incubation stage progress to the pre-AIDS stage at the rate ν_1.
- Infected individuals in the pre-AIDS stage progress to full blown AIDS at the rate θ.
- Individuals from all the compartments leave the population with the same rate as entry, that is b.
- Consider also the following two scenarios:
  1. Treated AIDS individuals remain in the AIDS stage.
  2. Treated AIDS individuals are assumed to achieve a very low viral load and a significantly high CD4 count and can therefore be considered to have made a transition to the incubation HIV stage with rate ν_2.

Tasks:

- Draw the compartmental diagram of disease dynamics.
- Write down the model equations.
- Derive R_0.

4 HIV infection in the liver: within-host model

The population is divided into seven compartments, namely uninfected CD4 cells (T_c), infected CD4 cells (I_c), uninfected hepatocytes (T_h), latently infected hepatocytes (I_f), actively infected hepatocytes (I_a), HIV-specific cytotoxic T-lymphocytes (L) and virus (V).

- A virus infects a hepatocyte with probability p or otherwise a CD4 with probability $(1-p)$.
- Uninfected CD4 cells T_c proliferate with a constant rate λ_1 and get infected in contact with the virus V with rate $\beta_1 V$.
- Uninfected hepatocyte cells T_h proliferate with a constant rate λ_2 and get infected in contact with the virus V with rate $\beta_2 V$.
- An infected hepatocyte will become immediately infectious (actively infected) I_a with probability q, or latently infected I_f with probability $(1-q)$.
- Infected CD4 and actively infected hepatocytes get killed by the immune cells with rate $k_1 L$.
- HIV-specific cytotoxic T-lymphocytes L proliferate in two ways: naturally from within-body production with constant rate x and in contact with infected cells with rate $k_2(I_c + I_a)$.
- Each infected cell I_c and I_a which raptures due to infection produces new virions with rates s_1 and s_2, respectively.
- Classes T_c, I_c, T_h, I_a, I_f, L and V die out/clear naturally with rates d_1, d_2, d_3, d_4, d_5, d_6 and d_7, respectively.

Tasks:

- Draw the compartmental diagram of disease dynamics.
- Write down the model equations.
- Derive R_0.